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Final Measurements of the Velocity of Light

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Improvements in the apparatus formerly used include the use of an eleven-stage electron multiplier tube, the control of the transmitter frequency by the laboratory standard, the substitution of a 1000-watt water-cooled mercury arc for the light source, a new type of Kerr cell, changes in the mirror supports and base line measurements, and the use of an automatic recorder. The apparatus has been completely rebuilt and converted to a.c. operation. The optical system has been changed to permit simpler measurements of the path difference, and electrical circuits have been devised to smooth out the fluctuations due to voltage variations and other causes.

Group velocity is discussed as a correction factor in this and previous measurements. The correction is shown to amount to as much as 7 km/sec. in some cases. Electron transit time is shown to be a limiting factor for this method of measuring the velocity of light. The final result of 2895 observations is given as 299,776±14 km/sec. This includes a group velocity correction and should not be compared with previous results without taking this into consideration. The conclusion is reached that the velocity of light is a constant as nearly as we can measure it at present.

INTRODUCTION

A SUMMARY of previous terrestrial measurements of the velocity of light and a brief discussion of the accuracy and method for each has been given by de Bray[1] and Mittelstaedt.[2] From this analysis, certain determinations were selected as presumably more reliable than others and a plot of these values against the mean time of their determinations suggested a possible linear decrease with time or a periodic function of time.[3] Birge and others[4] have discussed these apparent variations and their theoretical significance.

If, however, these various values obtained by direct measurement, as well as two obtained by indirect means, are plotted as lines instead of points, the length of each line representing the experimental error involved in each case, a graph as shown in Fig. 1 results. Under these conditions there will appear little reason for trying to find

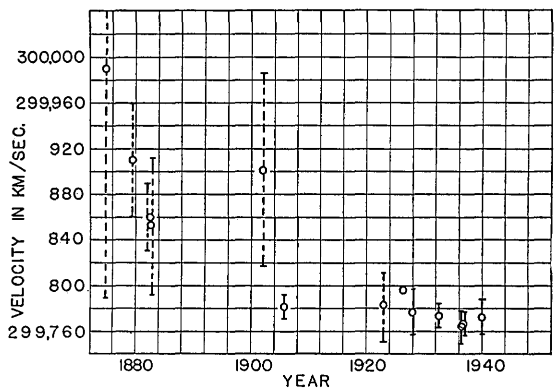

FIG. 1. Plot showing the principal measurements of the velocity of light by direct and indirect methods since 1870.

* Now with American Research Company, Beaumont, Texas.
[1] M. E. J. G. de Bray, Nature **120**, 602 (1927).
[2] O. Mittelstaedt, Physik. Zeits. **30**, 165 (1929).
[3] M. E. J. G. de Bray, Nature **133**, 464 (1934); F. K. Edmondson, Nature **133**, 759 (1934).
[4] R. T. Birge, Nature **134**, 771 (1934); Rev. Mod. Phys. **1**, 1 (1929); D. C. Miller, Rev. Mod. Phys. **5**, 3 (1933).

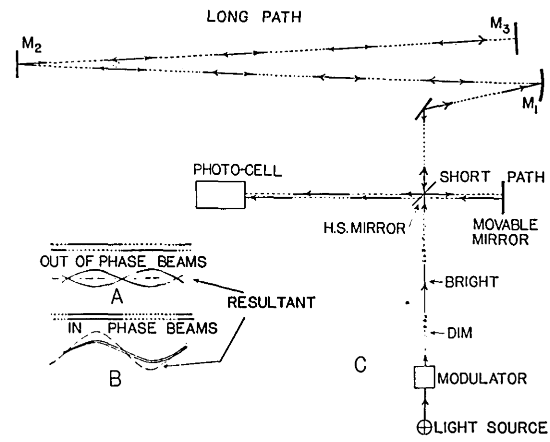

FIG. 2. Schematic diagram of the present method for measuring the velocity of light.

a variation of the same order of magnitude as the errors of measurement. Also the fact that the six most accurate determinations, including direct and indirect methods, and covering a period of 30 years, agree much more closely than is to be expected upon the basis of the periodic variations of Edmondson, or the linear variations of de Bray, seems to indicate that the variations in early measurements were due to experimental error rather than natural causes. Furthermore, it will be noticed that the short period variations found by Pease and Pearson[5] also were based upon statistical means and the actual spread of values is so great in comparison with the variations that an attempt to show periodicity seems somewhat unwarranted under the circumstances. Until sufficient evidence is forthcoming, i.e., until readings are sufficiently accurate, without statistical manipulation, to show variations in the velocity with time, it would appear somewhat anticipatory to assume, on data insufficiently accurate, that there are such. It was the purpose of the present assembly to attempt to settle this question one way or the other without too much dependence upon human observation and statistical manipulation.

A brief review of the method

The present arrangement[6] is best discussed with the aid of Fig. 2. A light beam is passed through a modulator where it is made to vary sinusoidally in intensity about some steady value. From the modulator the beam passes through a half-silvered mirror, a portion being reflected from the surface over to a movable mirror. From this mirror the beam is returned, passing through the half-silvered mirror to a photoelectric cell. The other portion of the original beam transmitted by the half-silvered mirror passes over the much longer path ($M_1M_2M_3$), and is returned along the same path, being reflected this time from the half-silvered mirror over to the same photoelectric cell. A tuned circuit converts these photoelectric currents into voltages which are amplified and recorded. It can be shown that the resultant voltage is dependent upon the phase relation of the original light modulations and this voltage will be either a maximum when the beams are in phase, or a minimum when one beam is an odd number of half-cycles behind the other. By noting the path difference for a given minimum position, the velocity of light is readily computed by the relation:

$$c = 2fs/n, \tag{1}$$

where n = the number of half-cycles phase difference, s = the optical path difference between the two light beams, f = the frequency of modulation of the beams, and c = the velocity of light.

APPARATUS

The transmitter

The light source formerly used was a 500-watt Pointolite. This was replaced by a 500-watt special projection lamp having an overlapping double row of filaments and later by a 1000-watt air-cooled lamp of the same type. The latter has a commercial rating of 27,000 lumens.

Since April of this year (1940) measurements have been made with a type H6 mercury vapor lamp. This is a high pressure, water-cooled capillary with a rating of 1000 watts and 65,000 lumens output. When this lamp was used it was unnecessary to use the 60-cycle modulation on the Kerr cell to obtain an audible component in the beam,[6] since the light has a decided 120-cycle modulation. Only the steady biasing voltage and the radiofrequency voltage were necessary.

The light is modulated by a water-cooled Kerr

[5] Michelson, Pease and Pearson, Astrophys. J. 82, 26 (1935).
[6] Cf. W. C. Anderson, Rev. Sci. Inst. 8, 239 (1937).

cell (see Fig. 3) similar to the ones previously used. The present cell has glass-sealed end plates and vertical cooling jacket to eliminate air bubbles when the cell is being filled. A polished ground-glass stopcock requiring no lubricant permits draining and refilling of the cell without removing its water and electrical connections. The entire cell is of Pyrex glass and was annealed for several hours in a furnace to eliminate strains. The electrodes are of polished nickel and were inserted after the end plates were sealed on to prevent oxidation. Very pure nitrobenzene is used for the liquid dielectric. A motor driven glass propeller circulates the liquid through the cell proper and the water jacket for cooling purposes.

The cell is operated with a biasing voltage of 5000 to 10,000 volts and about 90 watts of radiofrequency energy. From rough current measurements and reactance calculations this means that the radiofrequency voltage across the cell is about 1000 volts.

The radiofrequency voltage applied to the cell for modulation purposes is obtained from a power amplifier having an output of about 90 watts at a frequency of 19.2 megacycles. The driving voltage for the power tubes is secured by an arrangement as follows:

Voltage from the 50-kilocycle standard frequency generator of the laboratory, designed by Professor G. W. Pierce,[7] is amplified and then piped to the room housing the velocity of light apparatus. Here it is again amplified at 50 kilocycles and then used to drive a system of tuned radiofrequency amplifiers and doubling stages. After passing through eleven such stages, eight acting as doublers or triplers and three as straight amplifiers, the 19.2-megacycle driving voltage for the power tubes emerges.

The radiofrequency voltage is supplied to the Kerr cell by means of loosely coupling the coil of the cell tank circuit to the output coil of the power amplifier. Tuning of the Kerr cell circuit is accomplished by varying the position of a clip on the coil, and noting the illumination produced in a small neon bulb near the Kerr cell terminals.

[7] Cf. G. W. Pierce, Proc. Am. Acad. Arts Sci. **10**, 271 (1925); see also Proc. I. R. E. **16**, 1072 (1928).

The receiver and associated apparatus

The receiving system has been greatly improved over the one previously used. The various units with one or two exceptions have all been converted to permit operation from the regular alternating-current power lines instead of from batteries as formerly. It has always been a rather difficult problem to prevent interference from the stray radiation of the radiofrequency transmitter, only a few feet away. When the same alternating-current power lines are used for both the transmitter and the very sensitive receiving system, the problem becomes even more acute. In the past[6] a low-frequency component has been added to the light beam modulation to distinguish it from the unmodulated radiofrequency fields of the transmitter.

In the present arrangement, the power lines enter the shielded receiving system at only one point. Additional parallel lines are furnished the other units by means of shielded leads taken from an internal connection to this line. A four-section filter consisting of radiofrequency chokes in each

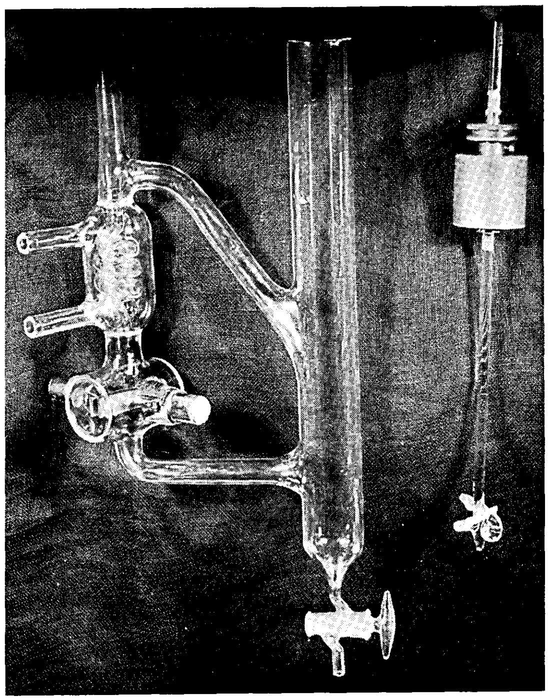

FIG. 3. Glass-sealed Kerr cell with stopcock for draining and refilling without removal from the apparatus. The water-cooling jacket and glass screw pump are similar to those previously used for this purpose.

line and condensers across the line is employed on the leads coming from the outside power lines, and similar two-section filters are used on all the secondary shielded leads to the other units. All power supplies, for both the transmitter and the receiver, are regulated to diminish voltage fluctuations.

The photoelectric cell formerly used has been replaced by a photosensitive electron multiplier tube of eleven stages. With 100 volts per stage the present tube has a sensitivity of about 2 amperes per lumen, compared with 30 microamperes per lumen for the ordinary vacuum photoelectric cell. This arrangement has the very desirable advantage of amplifying the light signal without doing likewise for the stray radio-frequency voltages picked up from the transmitter. Such is not the case for the stages following, but the light signal is given such an impetus by the multiplier tube that the gain in the following amplifier can be reduced to levels sufficiently low to practically eliminate the effect of any stray radiation.

The electrical circuit for a five-stage multiplier tube and the acorn pentode preamplifier stage is shown in Fig. 4. The five-stage tube was used in preliminary work and has since been supplanted by the eleven-stage one, but the circuit is very similar. The points marked A and B are connected to the input terminals of a sensitive receiver.

The audiofrequency output voltage of this

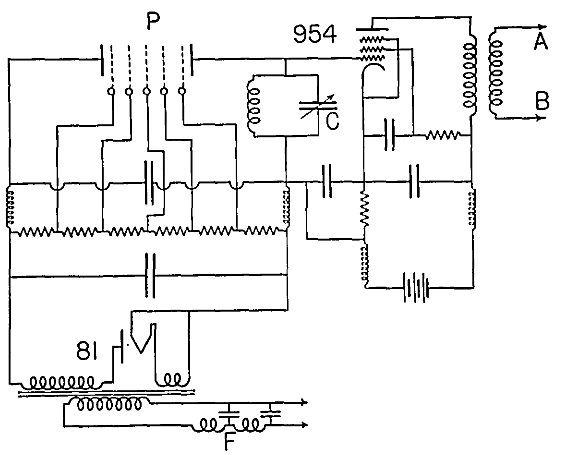

FIG. 4. Electron multiplier tube circuit and acorn pentode used for preamplifier stage. The terminals AB were connected to the input of a high gain short wave receiver.

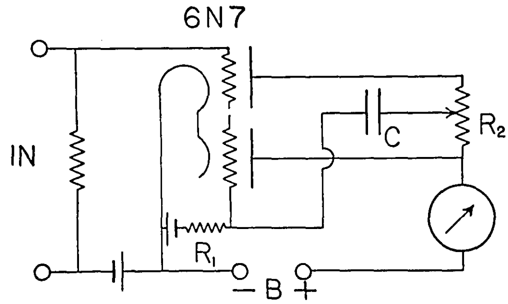

FIG. 5. Schematic diagram of feed-back circuit used for diminishing extraneous fluctuations in the output voltage.

receiver, after rectification, feeds into a circuit as shown in Fig. 5. In this circuit one triode acts as an ordinary direct-current amplifier. Fluctuations due to various causes will then be amplified by this tube as well as the slowly changing signal voltage. To neutralize these fluctuations a voltage from the output of the first tube is then fed back through the resistance-capacity coupling to the second triode. The plate circuits of the two triodes have a common path through the recording meter. It is easily seen that current variations having a period shorter than the R_1C factor of the feed-back circuit will be canceled in that portion of the circuit occupied by the meter. As long as the signal voltage varies slowly in comparison to the extraneous fluctuations and the R_1C factor, the resultant current will show the signal variations with greatly diminished fluctuations. Since the rate of change of the signal voltage is governed by the speed of the movable mirror, it is merely necessary to reduce the speed of the latter until the desired conditions are reached. The actual circuit used is shown in Fig. 6.

A photographic arrangement is employed to eliminate frictional errors usually associated with mechanical ink or waxed paper recorders. At first, lights illuminated the face of the meter (see Fig. 5) and an image of the latter was projected by means of a lens upon a narrow vertical slit. The image of the black meter needle traced a white line upon the paper because of insufficient exposure. The paper motion was governed by the motion of the movable mirror so that horizontal distances upon the record were directly mensurable. A fine vertical wire stretched across the paper holder served to produce a white

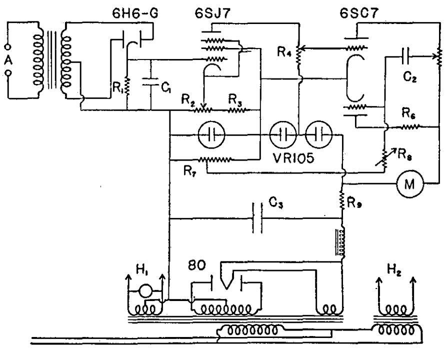

FIG. 6. Actual circuit used for fluctuation elimination.

fiduciary line on all records. Relays and trigger devices automatically limited the distance traversed by the moving mirror and recording arm. This arrangement has now been supplanted by a 35-mm motion picture camera adapted for the purpose. The face of the meter has been blackened and a small polished nickel wire placed beneath and at right angles to the needle and rigidly fastened to the face. When light strikes this surface it gives the equivalent of a narrow slit but with considerably more light than formerly was obtained. This arrangement in a box is placed at one side of the lathe bed. The camera is fastened to the movable mirror and the lens faces the meter face. A flexible bellows joins the two so that no stray light gets in (see Fig. 7). Horizontal as well as vertical motions are thus reduced to a size suitable for a frame $1'' \times \frac{3}{2}''$ on the paper. To determine the amount of reduction produced, the polished wire mentioned above is scratched at centimeter intervals. The motion of the mirror then shows these up as horizontal lines somewhat closer together, depending upon the reduction involved.

Optical arrangement

The optical arrangement is shown in Fig. 8. Light from the 1000-watt projection lamp passes through a condensing lens system, a Polaroid polarizer, the Kerr cell, and a second polarizer. At this point a second lens focuses the light upon a circular hole in a metal plate. The hole is traversed by fine cross hairs arranged to divide the opening into quarters. The purpose of the cross hairs will be discussed later.

After passing through the circular opening the beam strikes a half-silvered mirror, M_6, and is split. The reflected portion passes through a diaphragm, D, and a lens. The lens is placed at a distance equal to its focal length from the circular opening so that the rays emerging from the lens are essentially parallel. Hence the intensity or direction of the reflected light from the mirror, M_3, is not altered appreciably when the latter is moved through the necessary range for a reading. The light reflected from the mirror, M_3, retraces its path through the lens, and a portion passes through the half-silvered mirror to finally fall upon the photoelectric cell.

The diaphragm is used to match the intensity of the beam traversing the short path, with that of the beam passing over the long path. An exact match is not necessary as shown in a preceding discussion.[6] The more closely they are matched, however, the more accurately the minimum point can be determined.

The portion of the original beam which was transmitted by the half-silvered mirror passes over the path $M_6M_1M_5$.

From M_5 the beam is returned over the same path to the half-silvered mirror again. Here a portion is reflected and combines with the other

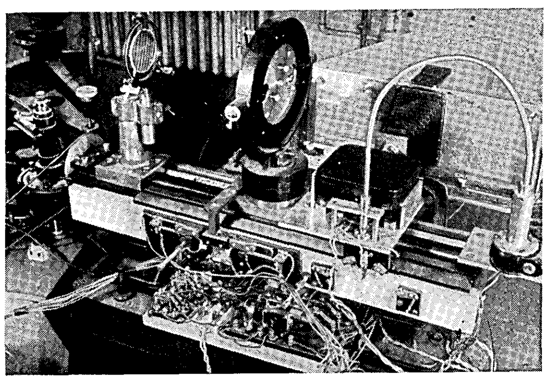

FIG. 7. Automatic recorder used in final phase of the work.

beam arriving from the short arm and the two together enter the photoelectric cell.

M_5 is an 8″ plane mirror. Mirrors M_1 and M_2 are concave; one is 9″ in diameter and has a focal length of about 5 meters; the other is 8″ in diameter and has a focal length of 2.5 meters. All mirror surfaces are accurate to within one wave-length of light.

The mirrors M_1 and M_2 are arranged in a dual-mounting assembly, back to back (see Fig. 9), so that either one can be rotated into proper focusing position with accuracy. The 9″ concave mirror is arranged to produce an image of the circular opening and cross hairs upon the surface of mirror M_5 at the end of the long path. The reflected light, in turn, retraces its path and forms an image the same size as the original just in front of the photoelectric cell surface. The image produced by light from the short arm also focuses at the same point and the two cross-hair images can be brought into exact coincidence by slight adjustments. The combined light then enters a very short focus lens and produces practically a point image upon the cathode of the photoelectric cell.

The 8″ concave mirror is arranged to reflect the light directly back over the path M_2M_6, without traversing the longer path M_1M_5. The focal length has been so chosen that it also produces an image at the same point as the light from the short arm. Thus either concave mirror can be rotated into position readily and adjusted so that the light either traverses the path $M_1M_5M_1M_6$, or the path M_2M_6. In order to produce a minimum with the 9″ concave mirror in position at say M_1, it is necessary that the path differences be equal to an odd number of half-wave-lengths of the radiofrequency modulation. That is,

$$2S+2x-2y=(2n+1)\lambda/2. \qquad (2)$$

With the 8″ concave mirror in position M_2, a minimum will be produced at some new position

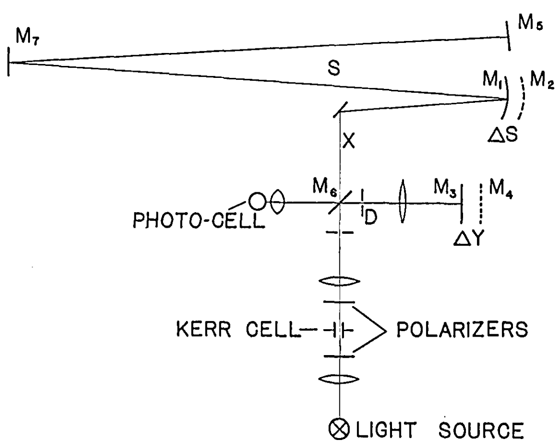

FIG. 8. Schematic diagram of the optical arrangement used for simplifying distance measurements.

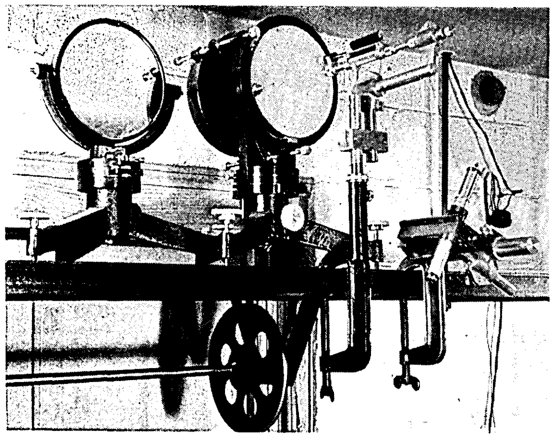

FIG. 9. Dual mounting for the concave mirrors (right) and single mounting for plane mirror (left). The micrometer microscope and special inside micrometer calipers for obtaining the mirror positions with respect to the tape are also shown.

of the lathe mirror, M_4. The relation in this case is

$$2x+2\Delta S-2y-2\Delta y=\lambda/2. \qquad (3)$$

If we subtract Eq. (3) from (2) we obtain the relation:

$$2S-2\Delta S+2\Delta y=n\lambda, \qquad (4)$$

where n is an integer. This means then that it is unnecessary to measure the length of the short path or the distances x and y. It is only necessary to know the length of the path S, and the short intervals ΔS and Δy. The distance measurements are in this way considerably simplified.

Control devices

The movable mirror is mounted upon a small lathe bed carefully machined to eliminate small surface irregularities. A millimeter-pitch screw is used to drive the mirror assembly. A pulley attached to one end is driven by an induction motor and serves to rotate the screw. A reduction-pulley arrangement enables various speeds of rotation of the screw to be obtained.

In Figs. 7 and 10 is shown the motor control assembly. A Bakelite strip A, attached to the moving mirror, tripped switches at either end and in the center of its path. The switches B and C, and relays D and E, stopped the motor, turned off the recording lights, L, and wound up a new section of film into place. The motor circuit was kept open for a sufficient time, about

20 seconds, to permit these various operations to take place before a relay closed a reversing switch and started the motor rotating in the opposite direction. The necessary lag was introduced by the thermal inertia of the heaters of the rectifiers shown in Fig. 10, which furnished the current to the relay coils F and G. Variation of the resistance R permitted control of the amount of thermal lag within certain limits.

The film-winding motor was controlled by means of special unidirectional switches operated by another moving arm mounted directly beneath the camera assembly (see Fig. 8). Single frame winding of the film was accomplished quite accurately by means of gears and a commutating device acting through another relay.

The time and date were automatically placed on each record by means of a special timing circuit and switch. Inside the recorder box directly beneath the recording meter but partitioned from it was a watch with reduced face and hands, and a slip of paper giving other data such as the date, etc. A small flashlight bulb was placed in front of these but shielded so that its direct light would not shine in the camera lens. An overvoltage was applied to the lamp briefly so as to give a "snapshot" of the paper and watch face on the film along with the record.

The greatest difficulty came in making the exposure time short enough "to stop" the motion of the lathe mirror and yet be consistent and automatic. This was finally solved by the use of the circuit shown in Fig. 11. The initiating impulse is caused by the closing of the switch S. This operates the relay K_1 to break the upper contact and close the lower, permitting the condenser C to discharge through the resistance R and applying a positive potential to the grid of a tube for a certain time interval determined by RC. If the tube was previously biased so that insufficient plate current flows through the relay K_2 to operate it, then sufficient current will now flow just as long as the abovementioned voltage applied to the grid of the tube exceeds a certain value.

It will be noticed that once the initiating key S is closed the rest of the operation is automatic. The interval that the relay K_2 is closed is determined by RC and not by the length of time S is closed, provided of course S is closed long enough for C to discharge. Thus the switch S can be any slow acting mechanical switch and yet the exposure time can be kept small and varied at will by adjusting the value of R.

Measurements

Frequency

The period of the radiofrequency modulating voltage was controlled by the primary standard of frequency of the laboratory as discussed earlier in this paper. The standard is a 50-kilocycle piezoelectric oscillator with the necessary temperature controls. It is checked frequently with the standard frequencies sent out from Arlington. A synchronous clock driven by a 1000–cycle subharmonic of the master oscillator is also checked with the national time signals. The error in time was certainly less than one part in a million and conceivably less than one part in eight million from all measurements made upon it.

Distance

The distance was measured with an invar-steel tape and special end-point apparatus. The tape was supported upon light 10″ pulleys with ball bearings and under a tension of 15 kg. A fine even-depth line was ruled upon each end of the tape. The distance between these lines was determined under the same conditions by the National Bureau of Standards. The error in absolute value was estimated to be less than one part in 210,000.

The distance between the mirrors was referred to the tape as a base line. This was done by

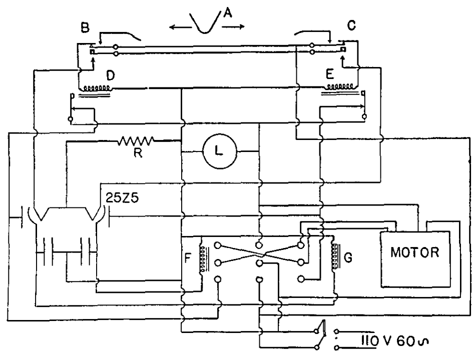

Fig. 10. Automatic motor control circuit for operating the recorder and lathe mirror.

means of a special device shown in Fig. 8. An inside micrometer caliper with calibrated rods was adapted for the purpose.

The micrometer is held so that the fixed end is against the side of an extremely fine bronze wire. The other end of the wire is weighted so that it hangs directly beside the ruled line on the tape. A micrometer microscope is then focused at the junction of the two and is used for measuring the small interval between the tape line and the side of the wire. This distance usually amounts to less than 0.2 mm and was read to 0.001 mm.

The opposite end of the inside micrometer has the movable head and the latter is rotated until it just touches the surface of the mirror at its center. A ratchet head was first employed but it was found that an electrical contact was better. A pair of high resistance headphones is placed in series with the micrometer and the aluminum surface of the mirror. A small alternating-current voltage is then imposed upon the arrangement. Readings can be repeated with this arrangement to better than 0.002 mm.

The tape is supported directly beneath each pair of mirrors and the distance measured as above. Twice the sum of these distances gives the total path length, S. The concave mirrors are interchanged and the difference in position measured.

The transference errors thus far have been less than 0.3 mm for the 171-meter interval.

The distance to be used in computation is the interval S plus the corrections ΔS and Δy. The interval Δy is obtained from measurements of the recorded minimum points for the concave

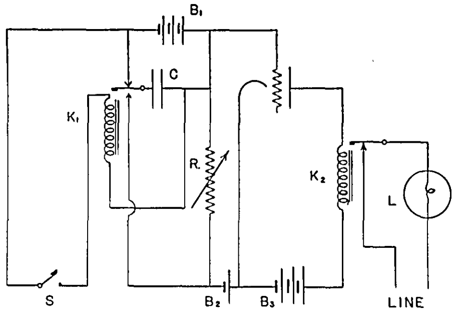

FIG. 11. Timing circuit for incorporating the date and time on a record by means of a brief exposure. The lamp L is flashed for a time interval independent of the length of time that the slowly acting switch S is closed.

mirrors interchanged. This distance is measured directly from the records with a comparator graduated in 0.01-mm units.

Procedure

The H.R.O. receiver is turned on at least a half-day before readings are to be taken and is usually left on continuously to prevent drift in the superheterodyne component due to a temperature rise. The other components are turned on several hours in advance and allowed to reach temperature equilibrium. This is particularly desirable for the Kerr cell.

With the electrical apparatus in thermal equilibrium, the light beams are made coincident by means of the cross-hair images.

The intensity of the stronger beam is then adjusted by means of the iris diaphragm until it equals that of the weaker beam as shown by the output meter reading.

With the camera loaded with 50 feet of 35-mm. film it is necessary to reload only about once a day. The recording apparatus is entirely automatic.

After a number of records have been taken with the first concave mirror in position, the other is rotated into position and records taken for it. This gives the long and short path intervals. Fig. 12 shows typical records.

Constant errors

In A of Fig. 13 is shown a cross-sectional view of an ordinary photoelectric cell with cylindrical shell type cathode and concentric wire anode, E. If A and B represent the points where the centers of gravity of two beams of light strike the cathode surface, then it is readily seen that the electrons ejected at A and B, neglecting initial velocity, will have equal transit times in going from the cathode to the anode.

In case B of Fig. 13, however, we have a different situation. This represents a cross-sectional view of the cathode and first few stages of an electron multiplier tube of the type used in this work. Here it will be seen that if the two centers of gravity of the beams C and D do not coincide, then the electron transit times will in general not be the same for the two beams. Thus if C represents the beam coming from the short arm and D that coming from the long arm of the

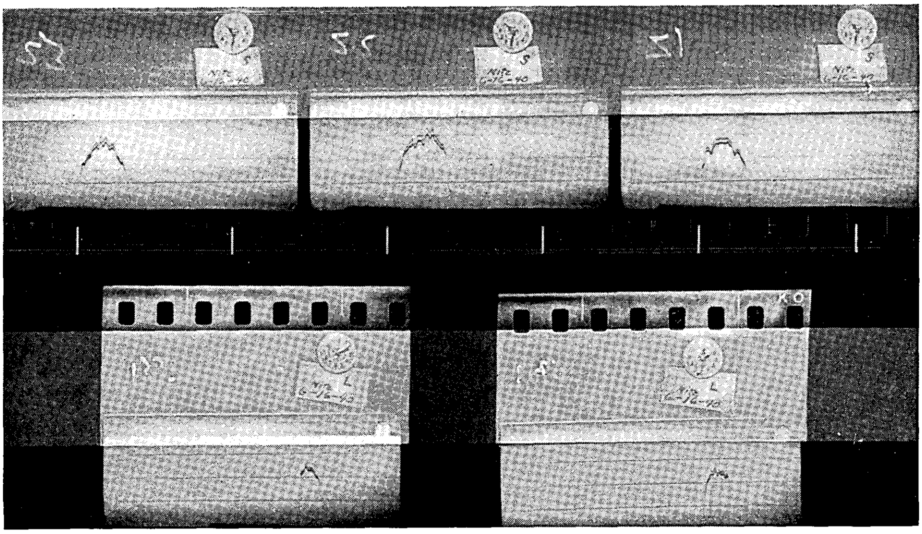

FIG. 12. Prints made from a series of records taken on the night of June 16–17, 1940. The upper ones are for the short path at about 10:30 P.M., while the lower ones are for the long path at about 2:00 A.M., June 17th. The original records are much clearer than these prints might indicate.

apparatus, there will be an *apparent* phase difference between the two *beams* that is due instead to a *real* phase difference between the *electrons* ejected by the two beams.

This factor has given considerable trouble and the variations shown in the results can very well be attributed to shifts in the coincidence of the two beams. This shift can be attributed to thermal expansion of the mirror holders in the long path, and to building vibrations. The former causes slowly changing shifts usually in one direction or the other during a run of several hours. The building vibrations, however, show up as rapid fluctuations and shifts. Even a steady wind can produce observable shifts in the minimum point.

Many different methods and devices have been tried to eliminate or minimize this error. These have included keeping the light beams as nearly coaxial and concentric as possible in the path leading to the photoelectric cell. Many weary hours of adjustment have been expended in this operation. A short focus lens placed next to the cell to reduce the size of the final images upon the cathode surface has also been used. At present the final images covered less than one millimeter and any variation in their coincidence was reduced about tenfold.

Rotating mixing devices have also been tried but usually introduced more error than they eliminated. The final solution in the author's opinion should be one of change in the design of the electron multiplier tube, but this brings up so many new factors that no satisfactory solution has been found to date.

Group velocity

A source of error apparently overlooked in all absolute determinations of the velocity of light for vacuum has been pointed out by Birge.[8] This is the correction for group velocity, assumed to be negligible by previous observers. Lord Rayleigh[9] and others[10] have discussed the effect of group velocity upon velocity measurements, but except for Michelson's carbon bisulfide measurements no other corrections appear to have been made for this phenomenon.

The expression for the group velocity is ordinarily given as:

$$U = V - \lambda dV/d\lambda$$

and the corresponding expression for the group

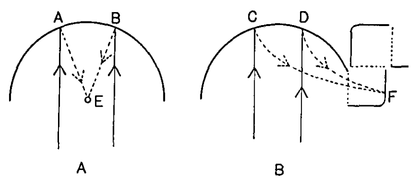

FIG. 13. Diagram showing in exaggerated form the effect of electron transit time on the final results.

[8] Private communication to the author.
[9] Lord Rayleigh, *Collected Papers* (1911), Vol. I, p. 322; Vol. VI, p. 41; Phil. Mag. *22*, 130 (1914).
[10] P. Ehrenfest, Ann. d. Physik **33**, 1571 (1910).

index of refraction is very nearly

$$\mu_g=\mu-\lambda d\mu/d\lambda.$$

Assuming the simple Cauchy expression for the dispersion of light in a medium,

$$\mu=A+B/\lambda^2+C/\lambda^4,$$

we obtain for the group index,

$$\mu_g=A+3B/\lambda^2+5C/\lambda^4.$$

The *variation* with wave-length for the group index is therefore over three times that for the wave index of refraction.

In the case of glass this correction becomes quite prominent and amounts to over three percent of the total correction. Even in Michelson's mile-long tunnel this factor is present due to the passage of the light back and forth through the window to the tunnel. Thick glass to withstand the pressure without deformation was probably used and amounted to several centimeters both ways.

In Mittelstaedt's experiment there was not only a correction because of the air and glass, but also for the nitrobenzene in the Kerr cells. He states that the change in optical path to vacuum increases the effective length by 21 mm due to lenses, 68 mm due to air, and 12 mm due to the nitrobenzene. If these corrections are in terms of wave velocity, then there will be an additional correction of about 1.8 mm for the glass, 2.4 mm for air, and 1.4 mm for nitrobenzene. The total additional correction to his value for the velocity of light because of group velocity, will then be approximately 6 km/sec. and make his value for c_0 approximately 299,784 km/sec. These computations are based upon assumed values as follows:

	A	B	C	λ	μ_g
Air				5500A	1.0002868
Glass	1.554	6.75×10^5	127×10^{10}	5500	1.628
Nitrobenzene	1.523	7.77×10^5	917×10^{10}	5500	1.650

The wave index of refraction for air as given by Meggers and Peters[11] is:

$$(\mu-1)\times10^7=2726.43+12.288/\lambda^2\cdot10^{-8}+0.3555/\lambda^4\cdot10^{-16}$$

with λ measured in angstrom units. The corresponding value for the group index is then:

$$(\mu_g-1)\times10^7=2726.43+36.864/\lambda^2\cdot10^{-8}+1.7775/\lambda^4\cdot10^{-16}.$$

[11] W. F. Meggers and Peters, Bull. Bur. Stand. **14**, 697 (1918).

TABLE I.

Date	Weight	Average c_0	Average Deviation: From Daily Mean	Average Deviation: From Final Mean
May 21, 1939	20	299,774 km/sec.	7 km/sec.	2 km/sec.
Nov. 8	17	299,775	9	1
13	35	299,759	10	17
15	79	299,772	9	4
16	140	299,780	8	4
27	103	299,781	13	5
Jan. 10, 1940	39	299,774	5	2
23	46	299,774	3	2
Mar. 4	30	299,757	3	19
7	56	299,745	9	31
8	257	299,754	3	22
11	147	299,749	9	27
Apr. 4	348	299,808	9	32
5	122	299,774	10	2
8	125	299,769	7	7
9	197	299,771	7	5
June 15	322	299,801	19	25
16	94	299,775	1.4	1
21	148	299,768	18	8
July 1	293	299,789	12	13
7	147	299,741	3	35
8	130	299,758	9	18
Final value	2895	299,776 km/sec.	±9 km/sec.	±14 km/sec.

In the previously published results[6] by the author this correction was also not applied and amounted to approximately 88 instead of 81 km/sec., making c_0 299,771 km/sec.

In the present case only the correction for air must be applied as there is no glass in the path to be measured. This correction amounts to approximately 84 km/sec. and is included in the correction factor in changing from c_a to c_0 in the results.

Results

The long path was measured seven times for an average value of 171.8642±0.0017 m exclusive of ΔS and ΔY. ΔS had an average value of 2.4770±0.012 cm. Therefore,

$$n\lambda=171.8147\text{ m}+2\Delta y.$$

Since $n=11$, and $f=19.2$ megacycles,

$$c_a=299{,}894.8+34.91\Delta y\text{ km/sec.},$$

where Δy is measured in centimeters. Inasmuch as the long path was slightly longer than an even number of wave-lengths, the lathe mirror moved in an opposite direction to that assumed in the derivation of the above equation, in order to produce a minimum. Hence Δy was always negative and the second term of the above equation for c_a was always negative.

A summary of the final results is shown in Table I. It will be noticed that while the average deviation from the daily mean is only ± 9 km/sec., the average deviation from the final mean is ± 14 km/sec. This is readily accounted for on the basis of the electron transit time difficulty mentioned above. The estimated error in the absolute value of the final result is therefore ± 14 km/sec. Until the transit time problem is overcome, this appears to be the limiting factor in the accuracy of this method.

In view of the above factor, no definite conclusions can be drawn regarding the periodic nature of the velocity of light, but inasmuch as this value checks very well with that of 299,771 km/sec. (group velocity correction applied) obtained by the author in 1937, and with the values of recent observers, the author is inclined to discount any probability of such a periodicity.

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