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CAN THE VELOCITY OF PROPAGATION OF RADIO WAVES BE MEASURED BY SHORAN?

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Abstract--The velocity of light and the velocity of propagation of radio waves are theoretically identical in vacuo. With refined methods of utilizing radar, long distances can be measured with considerable accuracy. This paper describes the methods used to measure 47 lines varying in length from 67 mi to 367 mi. Six of the lines measured could be compared with geodetic distances obtained from first-order triangulation. The entire network of lines was so designed that a rigid adjustment could be made. From the comparison with the six geodetic lengths and from an adjustment of the 41 other lines, a value of the velocity of propagation of radio waves in vacuo is deduced which exceeds by 16 km/sec the Anderson value of velocity of light in vacuo of 299,776 km/sec. All sources of error are discussed.

Introduction--The velocity of light has been determined from time to time since Cornu's experiments in 1872. DORSEY [1944] states "those values . . . . . have, in general, decreased monotously from Cornu's 300.4 megameters per second in 1874 to Anderson's 299.776 in 1940, the monotony being severely broken by the presence of Perrotin and Prim's 299.90 of 1902, between the adjacent values by Michelson, 299.853 in 1882 and 299.802 . . . . . in 1924."

Once again the monotony is being broken, but the more recent methods are the direct methods of the engineer rather than the laboratory methods of the physicist, and it is the velocity of propagation of radio waves which is being investigated. Theoretically, the velocities of propagation of radio waves and of light waves do not differ in vacuo, but the engineer prefers to measure directly the quantity that he wishes to determine. Utilizing a previously determined velocity of light, it may now be possible to use engineering methods of a simple nature to determine the velocity of propagation of radio waves by successive approximations.

The writer began his preliminary tests to utilize shoran to measure long distances with geodetic accuracy during World War II as a project of the United States Army Air Forces [ASLAKSON and RICE, 1946]. At about the same time the British noted that their observations with Oboe, a system comparable with shoran in accuracy, agreed better with triangulated distances if the Michelson value of 1935 (299,774 km/sec) were increased by 13.6. ESSEN [1947] deduced a value of 299,793 from a series of experiments. Essen measured the electrical resonance of a wave guide of short length and stated that his probable error, in what he described as a preliminary determination, was ± 9 km/sec. BERGSTRAND [1948] published a value of 299,796. Bergstrand used a "geodimeter" designed for direct distance measurement, utilizing the Kerr cell and photo tube principle. All three determinations mentioned above agree much better with Michelson's 1926 observations than with his 1935 work.

The British determinations during World War II were made by Jones and Cornford HART, 1948 by flying a Mosquito aircraft across an extended base line between two geodetic stations, as nearly as possible on a predetermined tracking range. Sea-level values were also observed over three sea paths of 1.9475, 31.4318, and 42.0960 mi from a station in North Devon. In the British experiments a mean linear lapse rate of the coefficient of refraction with altitude was assumed.

Also in Italy during the latter stages of the War, a distance of 618 km was measured by shoran. In this experiment the airplane was flown 22 times across the line between the ground stations. During each crossing the minimum distance was observed, and later reduced to sea-level distance. The altitudes were 11,000 and 15,000 ft. This work is described briefly by Hart who notes that if Essen's deduced velocity of 299,793 were used instead of the 299,776 for which the shoran computer is designed, the discrepancy would be reduced from +45 to +5 meters.

In the work under the direction of the writer which began with preparations and preliminary tests in 1943 and ended in 1947, refinements not made by Jones and Cornford and not used in the shoran measurements in Italy were incorporated in the observational, calibrational, and computational techniques. Studies of systematic errors were made and elaborate calibrational procedures were devised.

Although the observations agreed excellently among themselves, a systematic discrepancy with the geodetic distances between triangulation stations was noted. This lack of agreement largely disappears if a velocity of 299,792 km/sec is assumed. This is exactly the mean of the values of Jones and Cornford, Essen, and Bergstrand.

As stated above, V, the value of velocity in a vacuum, was taken as 299,776 km/sec in the instrument design. Converted to conventional units used in the United States, V is equal to 186,271.8 mi/sec. The principle of the shoran disign is satisfied for distance when the timing frequency in cycles/sec is equal to one half the velocity of electromagnetic propagation in mi/sec, or

$$f_t = V_t/2$$

where f_t = the frequency of the timing crystal and V_t = 1/2 travel time of the impulse in mi/sec.

The electromagnetic impulse actually travels through air of changing density and changing moisture content which change the velocity of propagation from point to point along its path. The velocity of propagation in air is given by

$$v = VK_1^{-1/2}$$

where v is the velocity of propagation in air, V is the velocity in a vacuum, and K_1 is the static dielectric constant of the air. K_1 varies from unity in empty space to K_s = 1.000567 for standard dry atmospheric conditions. The value of K_s adopted above is discussed later in this paper.

Therefore if V_s if considered the velocity of propagation of an electromagnetic wave through dry air under standard gas conditions, assuming V to be 299,776 km/sec,

$$V_s = 186{,}271.8/1.0002835 = 186{,}219.0 \text{ mi/sec}$$

The correct timing frequency should therefore be 93,109.5 cycles/sec if the above velocity is assumed. The calibration of this frequency is discussed later.

The design velocity of 299,776 km/sec was not seriously questioned by the project staff during the search for the systematic error with distance which continued to become more systematic with each refinement in calibrational and operational technique.

A comparison was finally made with three geodetic distances based on triangulation which had been executed by another organization. When these lines were measured, the discrepancies did not vary directly with distance. It has recently been revealed that the second group of triangulated lines contains a base-to-base length error of a magnitude that makes it unusable as a standard. This second network is now being reobserved with triangulation observations of a higher order. Meanwhile, the original shoran velocity data have once again been studied to ascertain the cause of the systematic error proportional to distance.

The problem was approached independently by two different persons. The writer based his investigation on a comparison of the shoran distances with the geodetic distances derived from the United States Coast and Geodetic Survey first-order triangulation, and it will be described first. The second method is described on pages 484 and 485

Before discussing the investigations, however, it is desirable to review very briefly the methods of observation when using shoran.

Methods of observation--In geodetic operations, shoran readings are used to determine the minimum sum of the distances from the airplane to two ground stations, and a scheme of triangulation is built up from these sum distances, no angles being measured (see Fig. 1).

The important refinements over previous shoran distance measurements were in the use of (a) a line-crossing technique designed to eliminate observer's errors, (b) least-squares computation of the minimum sum distance, (c) refined methods of determining the altitude of the airplane, (d) a velocity correction based on actual airplane weather reconnaissance at the time of the line-crossing measurements, (e) refinements in the geometric corrections for the reduction of the slant shoran distances to sea-level distances or to the approximate geodesic, and finally, (f) extensive instrument research which resulted in numerous modifications of the shoran system. The most important of these was the discovery of an error which was due to changes in the intensity of the signal and the design of a method to correct for this error. Many of the instrumental changes were suggested by

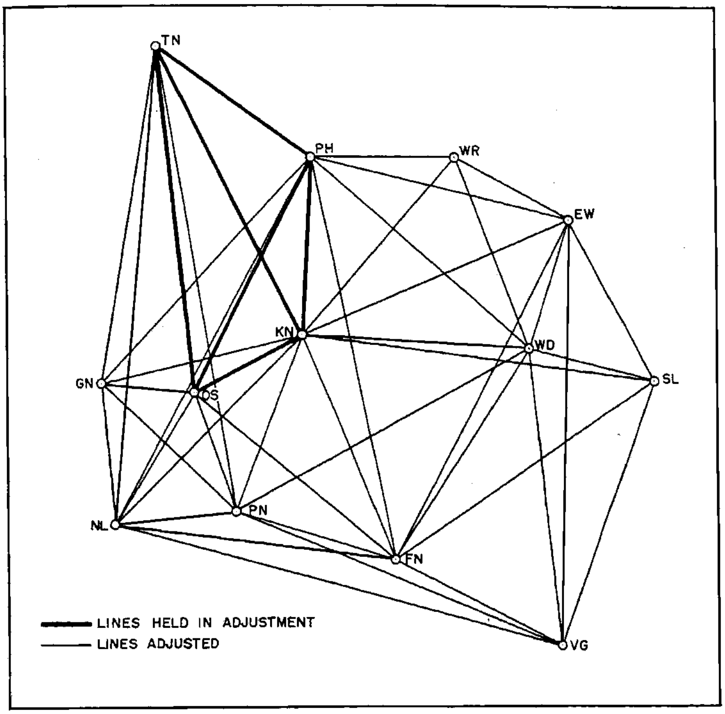

Fig. 1--Shoran network of 47 lines; the six heavy lines indicate those on which geodetic measurements were also made

Stuart W. Seeley of the Radio Corporation of America, the inventor of shoran.

A complete and detailed description of the methods is contained in a three-volume report which has not yet been released; the following material is, therefore, greatly abbreviated.

In an earlier paper [ASLAKSON and RICE, 1946], the preliminary measurement methods used at Denver were discussed. That work was experimental and is not considered here.

Line flying technique--The first step was to determine the minimum sum of two shoran distances from the plane to each of two ground stations as the plane crossed the line between the stations. The new technique was designated as the "Figure-8" line-crossing method. Four line-crossings made in the following manner constituted a "set of observations." All four crossings in a set were flown as nearly as possible at the same altitude and as nearly as possible halfway between the ground stations.

(1) The first crossing was made at approximately a 12°.5 angle to the normal.
(2) A second crossing was made from the opposite side of the line also at a 12°.5 angle to the normal but on the opposite side.
(3) The third crossing was made over about the same path as the second but in the opposite direction.
(4) The fourth crossing was made over the same path as the first crossing but in the opposite direction.

The new technique was effective in reducing the error caused by the observer's tendency to lag or lead in making a coincidence on the moving "pips." The Figure-8 method is analogous to the reversal method of observation in direction measurement with a theodolite and also to the spacing of the initial readings over various parts of the circle to eliminate errors due to eccentricities of the circle. Previously the lines had been crossed on a course normal to the line between the ground stations. In the original procedure the distances to the stations decreased until the minimum was reached and then began increasing. This caused the pips to slow down, come to a dead stop, and then change direction. The uncertainty of the observer as to when the exact minimum was reached resulted in ragged readings in the vicinity of the very critical minimum.

Using the Figure-8 line-crossing method, the pips continued to move at a nearly uniform rate throughout a crossing, and observations proved to be uniformly good

The form used for computing was also improved in flexibility. It was extended so that a varying number of observations could be used in computing a line-crossing. The time interval between observations was regulated by an intervalometer and was generally two seconds between successive frames. From 20 to 50 frames could be computed on the revised form, whereas in the preliminary work at Denver a fixed number of 31 frames was always computed. Inasmuch as the number of frames to be computed could now be varied, the exact number of intervals necessary to cover one mile of distance or one revolution of the goniometer was selected for computation. The troublesome goniometer error, sometimes called the "one-mile repeating error," was for all practical purposes eliminated from the result by this method of sampling the entire range of one complete cycle of the goniometer.

Altimetry--The altitude of the plane was determined with much greater accuracy than it had been during the preliminary work at Denver. Altitudes were computed independently from barometric altimeters and radio altimeters and the results verified the conclusion that any error in the reduced shoran distances due to an erroneous altitude was negligible. All instruments were carefully calibrated by rigid methods. The barometric altitudes were refined by observation of the lapse rate of the meteorological conditions by a second airplane or "weather reconnaissance plane." The weather observations were made at nearly the same time as the distance observations. Inasmuch as the weather observations were also used in the velocity correction which is discussed next, the entire flying procedure is described here. The weather reconnaissance plane began observations 500 ft above one of the ground stations, flew along the approximate shoran path to the point of line crossing, ascended to 1,000 ft above the highest altitude to be flown, and spiraled down to within 500 ft of the surface. It then spiraled upward to the height of the highest flight level and thereafter followed the approximate shoran ray path on a descending course to a height of 500 ft over the second ground station.

During the entire time psychrometric and time observations were made at 500-ft intervals up to the 5000-ft level, and at 1000-ft intervals above that level. The altitude of the shoran line-crossing plane was then computed from the following data: (1) Barometric altimeter and radio altimeter readings which were recorded photographically for each line-crossing. (2) Observed surface pressure and wet and dry bulb thermometer readings. (3) The intermediate psychrometric observations made by the weather reconnaissance airplane during the up-and-down spiral at the point of line-crossing.

The velocity correction--The psychrometric observations by the weather reconnaissance plane described in the preceding section were used to compute a velocity correction. In order to utilize the single parameter K to combine the effect of P, T, and e, the following relationship was employed

$$\Delta K \times 10^6 = 567 - 152.86\ P/T + 17.85\ e/T - 75.34e\ (T/100)^{-2} \quad \ldots\ldots\ldots (1)$$

where P = total pressure in millibars, e = partial pressure of water vapor in millibars, T = the absolute temperature (273°.16 C), and $\Delta K = (K_s - 1) - (K - 1)$. K_s is the dielectric constant of standard dry air (1.000567 at 760-mm pressure and 0°C) and K is the dielectric constant of the actual air as computed from (1).

For the small range of K encountered in shoran work the change in the velocity between any two conditions is a linear function of (K - 1). Therefore any deviation from the standard atmospheric conditions assumed in the construction of the shoran computer causes a velocity deviation that is directly proportional to the corresponding deviation in (K - 1) from the value (K_s - 1) assumed in the construction of the computer. The value of 1.000567 for standard dry air was based on the work of Hector and Woernley at the University of Buffalo in 1945 and was adopted instead of 1.000590 which had been used in the earlier work at Denver.

The value of the velocity correction ΔS was then determined from the following relationship.

$$\Delta S = (1/2)\int_0^S \Delta K\, dS \quad \ldots\ldots\ldots (2)$$

A numerical integration procedure was devised to compute the velocity correction, using the values of ΔK computed at appropriate intervals from the weather reconnaissance plane data and increments of the distance at the corresponding points along selected ray paths which approximated the actual ray path. Extensive use was made of tables, particularly "Ray-path tables" from which the distance increments needed in the numerical integration could be quickly interpolated for any path that would actually be flown.

During the preliminary work at Denver a "standard atmosphere" for the area was assumed and the velocity corrections were obtained from tables computed for that standard. Thus the new technique was a decided improvement over the original precedure.

Geometric corrections--The relationship employed to compute the reduction of the shoran distance to the sea-level or map distance was refined to introduce small tabulated corrections for all approximations made in the development. It has been thoroughly investigated and can be considered as rigid.

Instrumental research and corrections--Extensive research in shoran measurements made over a two-mile base with a ground station on an extension of the base and at a surveyed distance from its end, furnished valuable information which enabled many errors of the original shoran bombing instruments to be removed or greatly minimized by electronic modifications. The ground stations were also calibrated over the base, several calibrations of each ground station being made during the progress of the project. The timing crystals were also calibrated to an accuracy of the order of one part in one million.

The Anderson value of velocity was incorporated in the shoran computer by the use of a crystal with a frequency of 93,109.5 cycles/sec or one-half of the velocity of light (Anderson) of 186,219.0 statute mi/sec at 0°C and 760 mm pressure (dry air). The calibration determined the crystal frequency to be 93,109.65 cycles/sec and the difference of 0.15 cycles/sec was used to correct the results according to the following relationship:

$$\Delta S_t = S_t \Delta f / 93{,}109.5 \quad (3)$$

where ΔS_t = the correction to the shoran distance due to the crystal error, S_t = the measured shoran distance, and Δf = the above difference of 0.15 cycles/sec. The crystals were calibrated by the standard frequencies of the National Bureau of Standards' broadcast over WWV.

The airborne zero correction (essentially an index correction) and the ground-station calibration correction were applied to each shoran distance. Other investigations too numerous to describe here resulted in many modifications which improved instrumental accuracy.

Signal-intensity correction--The somewhat erratic variations in the results obtained on the Denver project were not understood until after the completion of that project. As the result of a conference between Carl Jacobsen and William Adkisson, the electronics engineers of the Project Staff, and Stuart W. Seeley of the Radio Corporation of America, the conclusion was reached that there was an error of considerable magnitude in shoran measurements arising from the fact that the slope of the received pulse varied with signal intensity. A timing error which was a function of signal strength was strongly indicated. A long series of tests proved this to be true, but the problem of elimination seemed insurmountable. Redesign of the equipment was impossible in the alloted time. The comparatively good final results of the Denver project were apparently due to the fact that so many measurements were made over an extended period of time that the signal-intensity error was partly eliminated by multiple observations under many varying conditions of signal intensity. But to be economical and practical in a regular field project, the number of observations had to be reduced. Inasmuch as the instrument could not be redesigned in the time available, a means of correcting for the signal-intensity error was sought.

A standard Air Forces instrument used to observe relative signal strength was modified and calibrated. The signal intensity, I, was observed for each observation. In the southwest corner of the scheme (see Fig. 1), where the first observations were made, only one signal intensity to each ground station was observed for each set of Figure-8's. Later the equipment was modified so that the signal intensity from each station could be observed simultaneously with the line measurement, thus making eight intensity observations for each set of Figure-8's. The fewer intensity measurements in the early part of the project may account for the larger corrections which are found in the southwest corner of the scheme and which are discussed later.

A total of 139 values of I were observed. Each pair of values at any given altitude gave a slope at the approximate mid-point between the two intensity values. Thus, an empirical relationship between error and signal intensity could be determined by least squares. The following equation resulted:

$$I.\,C. = K + 0.10629\,I \quad (4)$$

where I. C. is the signal-intensity correction expressed in units of 0.0001 and K is a constant for zeroing the curve.

The value of K was determined by two methods and was found by each method to be so close to zero as to be negligible. One method of determining K was to place a constant term in the least-squares adjustment of the triangulation network for each line. The second method was called the "sliver-triangle analysis" and is discussed later.

The spread of the observations was materially reduced by applying the signal-intensity correction and this was accepted as evidence of its validity. The intensity corrections were not related to the length of line but were a function of the altitude of the plane and the distance from the line joining the stations. Actual surveys of the intensity lobes were made at several times and they showed that the shape of the lobes conformed very closely with theory. In fact, a method of predicting the altitude to fly to produce given signal intensities proved very successful. In this system of prediction, the altitudes of the airplane and ground stations, the distances from the airplane to the ground stations, and the type of reflecting surface were considered. Jacobsen developed simple nomograms for making the predictions.

Sliver-triangle analysis--If there is a constant error K in any measurement and a distance is measured as a whole and also in two parts, the sum of the latter two measurements will differ from the whole measurement by the constant error K. Similarly in the case of any very long, thin, or sliver triangle found in shoran, the short sides can be projected onto the long side and value of K can be determined. The relationship may be expressed as follows.

$$K = \Delta L/[1 - (\cos A + \cos B)] \quad \ldots\ldots\ldots\ldots (5)$$

where A and B are the acute angles and ΔL is the difference between the long side and the sum of the two short sides projected onto the long side. For the purpose of this study, a large number of sliver triangles were inserted in the shoran scheme because they impose a most rigid condition in the adjustment, a fact quite contrary to ground triangulation. Sixteen such triangles were analyzed with the following result: K (by sliver triangles) = + 0.0008 mi; K (from adjustment) = + 0.00004 mi. Inasmuch as the probable error of each value was slightly greater than the value of K it was assumed that any constant error was negligible and K was accepted as zero.

It should be noted that a sliver-triangle analysis serves somewhat the same purpose as triangle closures in conventional triangulation, as an approximate check in detecting faulty observations during the progress of the field work.

Results of comparison of shoran measurements with inversed first-order triangulation distances--Using the methods described in the preceding paragraphs all six lines of a quadrilateral and its diagonals were measured by shoran. The results are tabulated in Table 1.

It may be noted that a complete set of observations by the Figure-8 method, or four crossings in all, must be used as a unit. All observations in a set must be made by the same operator. The observational procedure thus removes the personal equation of an observer, such as leading or lagging behind the pip during coincidence. With three sets of observations an average of between 700 and 800 individual distance readings are utilized. Each operator also reads his own zero, so the personal equation tends to be eliminated. A marked difference in results between observers was sometimes noted on the preliminary project at Denver, but an elaborate study of the later project failed to show any such difference. The consistency between sets is remarkable, the maximum deviation from the mean being 0.0025 mi or about 0.0042 km. The mean deviation is ± 0.0013 mi or 0.0021 km. The numbers 4.2 and 2.1 represent deviations of observational results in the sixth digit and may be compared with the 35 and 14 in Table 2 which lists Anderson's observations. It seems justifiable to conclude that the shoran observations were more uniform than Anderson's observations.

Furthermore, the means of sets of shoran observations repeated on different days showed smaller differences than individual sets observed on the same day. This is illustrated in Table 3 which lists all lines on which measurements were made on different days. It will be noted that excepting the single observation on January 22, 1947 on the line PH-FN, the agreement between the results on different days is excellent. The mean range or spread is 0.0014 mi. This indicates close agreement regardless of whether the observations were repeated on the same days or on different days.

The internal consistency of the shoran observations being of such a high order, it was a shock to the writer and his staff to find such large differences between the shoran distances and the geodetic distances computed from United States Coast and Geodetic Survey triangulation as listed in

Table 1--Shoran measurements of geodetic lines

Line	Designation	Identifying altitude[a]	Distance[b]	Signal Intensity Correction	Corrected Distance	Mean	Deviation from mean
		ft	mi	mi	mi	mi	mi
1	OS-KN	2,300	101.9151	0.0102	101.9253		-0.0012
		2,500	101.9161	0.0111	101.9272		+0.0007
		2,600	101.9180	0.0091	101.9271	101.9265	+0.0006
2	KN-PH	12,000	139.0453	0.0110	139.0563		-0.0007
		3,600	139.0516	0.0079	139.0595		+0.0025
		4,100	139.0452	0.0125	139.0577		+0.0007
		5,700	139.0402	0.0144	139.0546	139.0570	-0.0024
3	TN-PH	4,000	143.5808	0.0094	143.5902		-0.0009
		6,300	143.5748	0.0177	143.5925		+0.0017
		7,900	143.5755	0.0150	143.5905	143.5911	-0.0006
4	OS-PH	10,700	208.7168	0.0162	208.7330		+0.0014
		8,800	208.7151	0.0148	208.7299		-0.0017
		8,000	208.7201	0.0118	208.7319	208.7316	+0.0003
5	TN-KN	16,400	227.1233	0.0109	227.1342		+0.0012
		13,300	227.1253	0.0065	227.1318	227.1330	-0.0012
6	TN-OS	22,400	235.6500	0.0064	235.6564		-0.0020
		23,100	235.6536	0.0067	235.6603	235.6584	+0.0019
			Average deviation from mean ± 0.0013				

[a]In computations, a more exact altitude was used for each line crossing.
[b]Distance, based on Anderson's value of 299,776 km/sec, reduced to geodetic or sea-level distance with all corrections applied except signal-intensity correction. Each distance listed is the mean of four line crossings of the Figure-8 method and each line crossing utilized about 30 recorded distances for each station, or about 60 distances in all. Thus about 240 readings, read from 35-mm photographic-recorder film, usually photographed at two-second intervals, enter into each distance listed.

Table 4. It was noted that the differences tended to be proportional to distance but, as stated in the Introduction, three additional measured lengths connected to the triangulation of another organization failed to show the same correlation. The shoran measurements, of course, were questioned. It is now certain that the triangulation containing the doubtful connections was of too low an order to be used as a standard, so another investigation was made to ascertain the cause of the systematic error. The writer was much encouraged by the recent work of Essen and Bergstrand and by the fact that the British on the basis of the limited Oboe experiment did not hesitate to question the Anderson value.

Discussions of error--In tracing the source of the systematic error, the first point to note is that it is evidently at least approximately proportional to the distance. This can be seen in Table 4, where the residuals tend to be proportional to the lengths. (It should be noted further that the smallest of these residuals is more than four times the average deviation of an individual line measurement from the mean.) The same conclusion can be reached from the other lines of the figure not common to Coast and Geodetic Survey triangulation, if we bear in mind that the effect of a change in velocity is to change the scale of the figure. It follows that an erroneous velocity will not introduce any internal discrepancies in the figure, except in the neighborhood of ties to fixed points. On the other hand, an erroneous constant correction to all lengths will introduce closures, especially in the sliver triangles. The failure to find any significant value of K, either from the sliver triangles or from the general solution, indicates that no appreciable systematic error in the form of a constant affecting all lengths can exist in the present techniques. Furthermore, a constant error larger than 0.0036, even with different signs for different stations, is extremely unlikely to exist in the solution, for this value represents the total probable error deduced from the least-squares solution. This amount is roughly one third of the average discrepancy of 0.0094 found. We conclude that the error must be at least roughly proportional to the distance and that no significant constant error exists.

Table 2--Table of Anderson's values of 1939-1940[a]

Date		Weight	Mean velocity in vacuum	Average deviation from mean	
				Daily	Final
			km/sec	km/sec	km/sec
1939					
May	21	20	299,774	7	2
Nov.	8	17	299,775	9	1
	13	35	299,759	10	17
	15	79	299,772	9	4
	16	140	299,780	8	4
	27	103	299,781	13	5
1940					
Jan.	10	39	299,774	5	2
	23	46	299,774	3	2
Mar.	4	30	299,757	3	19
	7	56	299,745	9	31
	8	257	299,754	3	22
	11	147	299,749	9	27
Apr.	4	348	299,808	9	32
	5	122	299,774	10	2
	8	125	299,769	7	7
	9	197	299,771	7	5
June	15	322	299,801	19	25
	16	94	299,775	1.4	1
	21	148	299,768	18	8
July	1	293	299,789	12	13
	7	147	299,741	3	35
	8	130	299,758	9	18
		2,895	299,776	9	14

Estimated dubiety ± 14

[a]See DORSEY [1944].

An error varying with the distance could be attributed to any of the following causes: (1) A scale error in the goniometer, which measures the time taken for the pulse to travel the distance. (2) A scale error in the first-order triangulation of the United States Coast and Geodetic Survey. (3) An error in the meteorological data or its mathematical use. (4) An error in the physical constant employed to compute the dielectric constant of air. (5) An error in the signal-intensity correction. (6) An error in the velocity of propagation.

Under (1) the goniometer measures the time of travel of the pulse by measuring the phase difference (in terms of a vibration of 93,109.5 cycles/sec) between the instant when pulse A returns to the airborne set and the instant when pulse B is sent out. Between pulse A and pulse B, a large number of other pulses have been sent out, a number, in fact, equal to the number of statute miles between the ground station and the airborne station, provided that the crystal is accurately tuned. The goniometer also counts these pulses. Thus the possible errors of the goniometer are in three parts: first, those due to an error in the pulse repetition rate; second, the error of counting pulses; and third, a periodic error, called the "one-mile goniometer error." The counting errors are practically negligible; and the one-mile periodic error, which is quite appreciable, was eliminated, first by direct calibration on a known ground course, and second by an oblique line-crossing technique which spread the readings on both goniometers completely around the dial, so that they averaged out. In any case, no periodic error of this type could be mistaken for an error proportional to the distance. The pulse repetition rate is controlled by the airborne crystal oscillator. This, in turn, was constantly checked by a temperature-controlled crystal at the ground station, which was repeatedly calibrated against WWV. It is generally considered that errors with this technique should not exceed one part in one million.

With regard to the second source of error cited, reference is made to Special Publication No. 108 of the U. S. Coast and Geodetic Survey, which contains a discussion of the errors of the triangu-

Table 3--Shoran measurements repeated on different days

Name	Date	Altitude	Shoran distance	Mean	Difference between means
		ft	mi	mi	mi
EW-SL	Mar. 8, 1947	3,800	147.8680		
		4,300	147.8649		
		5,700	147.8610	147.8646	
	Mar. 14, 1947	3,800	147.8694		
		4,400	147.8645		
		5,800	147.8606	147.8645	+0.0001
PN-VG	Jan. 31, 1947	13,400	278.9814		
		14,000	278.9867	278.9840	
	Feb. 19, 1947	15,800	278.9835	278.9835	+0.0005
NL-PH	Dec. 14, 1946	21,400	331.3051	331.3051	
	Dec. 17,1946	23,000	331.3070		
		24,400	331.3050	331.3060	-0.0009
NL-KN	Jan. 8, 1947	11,100	214.7972		
		11,500	214.8068		
		11,900	214.7892	214.8007	
	Jan. 22, 1947	12,000	214.8040	214.8040	-0.0033
GN-KN	Dec. 13, 1946	5,900	163.2812		
		3,900	163.2849		
		8,000	163.2793	163.2818	
	Jan. 14, 1947	5,900	163.2823	163.2823	-0.0005
PH-FN	Jan. 22, 1947	20,700	321.0300	321.0300	
	Jan. 23, 1947	24,100	321.0341	321.0341	-0.0041
	Jan. 28,1947	28,250	321.0357	321.0357	-0.0016
KN-PH	Dec. 11, 1946	12,000	139.0563	139.0563	
	Jan. 15, 1947	3,600	139.0595		
		4,100	139.0577		
		5,700	139.0546	139.0573	-0.0010
KN-WD	Feb. 8, 1947	4,100	181.0109	181.0109	
	Feb. 18,1947	7,000	181.0127		
		8,400	181.0064	181.0096	+0.0013
Average difference between means of sets of measurements of the same line made on different days					0.0014

lation in eastern United States. It is shown that no section of this triangulation in the general adjustment required a correction greater than one part in 66,000; and that 43 out of 55 had a closure better than one part in 100,000. It is not possible that the Coast and Geodetic Survey triangulation has a scale error of 1:20,000 as a whole.

Concerning the third source of error the whole mathematical procedure of velocity corrections and geometrical corrections was independently investigated by KROLL [1949] who integrated the basic differential equation of terrestrial refraction along the ray path, thus obtaining both the velocity corrections and the geometrical corrections simultaneously by methods which were mathemati-

Table 4--Shoran distances based on Anderson's velocity (299,776) compared with geodetic distances

Line	Geodetic distance	Reduced shoran distance	Difference[a]	Difference divided by geodetic distance
	mi	mi	mi	
1	101.9344	101.9265	-0.0079	0.000078
2	139.0629	139.0570	-0.0059	0.000042
3	143.5987	143.5911	-0.0076	0.000053
4	208.7428	208.7316	-0.0112	0.000054
5	227.1442	227.1330	-0.0112	0.000049
6	235.6710	235.6584	-0.0126	0.000053
		Mean	-0.0094	0.000055

[a]Shoran distance minus geodetic distance

cally independent of those of the staff working on shoran and which led to sensibly different paths. Kroll's final reduced shoran values agreed within a few feet, however, with the values produced by the project formulas. Kroll further concluded from his analysis that no significant contribution to the error could arise from errors in the meteorological measurements.

Concerning the fourth cause of error it should be noted that the dielectric constant for air consists of two parts: first, that for dry air; and second, the contribution due to moisture. The contribution due to dry air is derived from the optical index of refraction, which is accurately fixed from astronomical refraction measurements. That for moist air is somewhat uncertain; but the correction required to bring the measured distances into conformity is far larger than the physical uncertainties.

Concerning cause (5) it will be noticed from Table 1 that, contrary to what might be expected, the intensity correction is not correlated with the distance. This is a consequence of the choice of position in the lobe pattern made in flying the shoran lines. It was generally true throughout the operation that the intensity was uncorrelated or very weakly correlated with the distance. The correlation was certainly not strong enough to make any apprciable contribution to the error.

Nevertheless, it should be admitted that the effect of an error in the signal-intensity correction upon the resulting velocity would be very considerable. A least-squares solution by Kroll shows that if the signal-intensity correction is set equal to zero, a velocity is obtained almost precisely equal to Anderson's velocity with no appreciable change of the probable error. On the other hand, a value of zero for the signal-intensity correction seems physically implausible. The value of 0.00001 mi per unit of intensity was obtained from 139 observations and has a probable error of one-tenth of itself. The intensity correction thus appears to be quite well established, but further study of this point is indicated.

We are thus left with (6), the velocity of propagation of electromagnetic waves in vacuo, as the most probable source of the discrepancy.

Derivation of velocity--In accordance with the above reasoning the writer employed the following method to ascertain a value of velocity which would remove the correlation of error with distance. Increments were deduced which, added to the Anderson value of 299,776 km/sec, furnished new velocities which would best fit the goedetic distances, according to the following relationship:

$$\Delta V = (\Delta S/S)\, V_I \qquad (6)$$

where ΔV is the velocity increment to be added to V_I (299,776 km/sec),ΔS = the difference between shoran and geodetic distances, as listed in Table 4, and S = the reduced shoran distance. The tabulation is shown in Table 5.

The derived velocity of 299,792.4 km/sec agreed closely with Essen's 299,793, Bergstrand's 299,796, and Jones and Cornford's 299,788, being almost the exact mean of the three values.

Discussion of this new value of velocity with officials of the Army Map Service revealed the fact that an independent investigation has been made in that office using a different method. In their investigation, the geodetic distances were held fixed and the remaining 41 lines of the shoran network

Table 5--Computation of velocity from geodetic distances

Line	Difference[a]	Correction ΔV	Corrected velocity $V_I + \Delta V$	Deviation from mean
	mi	km/sec	km/sec	km/sec
1	-0.0079	+23.2	299,799.2	+6.8
2	-0.0059	+12.7	299,788.7	-3.7
3	-0.0076	+15.8	299,791.8	-0.6
4	-0.0112	+16.1	299,792.1	-0.3
5	-0.0112	+14.8	299,790.8	-1.6
6	-0.0126	+16.0	299,792.0	-0.4
		Mean	299,792.4	

[a]From Table 4

(see Fig. 1) were adjusted. In the adjustment, in addition to the K term, an additional term was introduced to give a multiplier by which all lines would be changed in length. The Rice adaptation of the variation of coordinates method was used by Mary Jane Camplair of the Army Map Service in making this least-squares adjustment.

The velocity resulting from the Camplair adjustment was 299,792.3 km/sec and K was practically zero. This was considered as strong evidence that the error was systematic with distance, applied equally well in all parts of the network, and that there was no appreciable constant error. The distances resulting from applying the velocity of 299,792.3 km/sec to the six lines on which geodetic measurements were made are given in Table 6, together with new comparisons with the geodetic distances.

Table 6--Comparison of geodetic and corrected shoran distances

Line	Geodetic distance	Corrected shoran distance	Difference
	mi	mi	mi
1	101.9344	101.9320	-0.0024
2	139.0629	139.0646	+0.0017
3	143.5987	143.5989	+0.0002
4	208.7428	208.7429	+0.0001
5	227.1442	227.1453	+0.0011
6	235.6710	235.6712	+0.0002
	Mean with regard to sign		+0.00015
	Mean without regard to sign		0.00095

The corrections or so-called v's derived from the adjustment are listed in Table 7. Column 4 lists the v's when a velocity of 299,776 km/sec is used and column 5 lists the v's when 299,792.3 is used.

Discussion of shoran results--It may be noted in Table 7 that Lines 7, 29, 30, 39, and 46 show very large decreases in the v's when the deduced velocity is used. Two lines (8 and 45) show moderate increases.

The probable error derived from the adjustment when using Anderson's velocity is ± 0.00402 statute miles and is reduced to ± 0.00355 when the adjustment is made with the deduced value of 299,792.3 km/sec.

Of a total of 47 lines, including the measurements over fixed lines, the ratio of the error (or the v) to distance on 43 lines is 1/25,000 or better. The remaining four lines, namely lines 28, 30, 37, and 43, have ratios of 1/14,300, 1/24,000, 1/18,100, and 1/20,800, respectively. The reason for the larger v's in the southwest part of the scheme may be due to the fact that the observational technique for observing signal intensity was not fully developed at the time those lines were flown.

It is also possible that somewhat of a "'squeeze" occurred by holding all four Coast and Geodetic Survey stations fixed. As a test of shoran, it might have been somewhat better to apply the

Table 7--Corrections derived from the adjustment of 41 measured lines

Line	Designation	Approx. Length	Corrections in adjustments V = 299,776	Corrections in adjustments V = 299,792.3	Numerical changes in corrections
		mi	mi	mi	mi
7	TN-PN	336	+0.0077	+0.0012	-0.0065
8	TN-NL	340	-0.0024	-0.0059	+0.0035
9	TN-GN	229	-0.0009	-0.0019	+0.0010
10	WR-EW	102	+0.0008	+0.0010	+0.0002
11	WR-WD	163	-0.0037	-0.0032	-0.0005
12	WR-KN	186	+0.0041	+0.0033	-0.0008
13	WR-PH	112	-0.0034	-0.0024	-0.0010
14	EW-SL	148	+0.0020	+0.0017	-0.0003
15	EW-VG	344	-0.0014	-0.0017	+0.0003
16	EW-WD	108	-0.0007	-0.0001	-0.0006
17	EW-FN	304	-0.0001	-0.0001	0
18	EW-KN	230	+0.0014	+0.0016	+0.0002
19	EW-PH	206	-0.0008	-0.0014	+0.0006
20	SL-VG	225	+0.0014	+0.0013	-0.0001
21	SL-FN	246	-0.0003	-0.0007	+0.0004
22	SL-KN	278	+0.0020	+0.0030	+0.0010
23	SL-WD	99	-0.0032	-0.0038	+0.0006
24	VG-NL	367	-0.0016	-0.0027	+0.0011
25	VG-PN	279	+0.0020	+0.0021	+0.0001
26	VG-FN	148	+0.0002	+0.0013	+0.0011
27	VG-WD	241	-0.0003	-0.0002	-0.0001
28	NL-GN	113	+0.0090	+0.0079	-0.0011
29	NL-PH	331	+0.0060	+0.0024	-0.0036
30	NL-OS	123	-0.0127	-0.0051	-0.0076
31	NL-KN	215	-0.0012	-0.0003	-0.0009
32	NL-PN	97	+0.0016	+0.0016	0
33	NL-FN	226	+0.0049	+0.0035	-0.0014
34	GN-PH	243	+0.0072	+0.0060	-0.0012
35	GN-KN	163	-0.0006	-0.0007	+0.0001
36	GN-OS	67	+0.0001	+0.0020	+0.0019
37	GN-PN	145	-0.0068	-0.0080	+0.0012
38	PH-WD	227	-0.0040	-0.0063	+0.0023
39	PH-FN	321	+0.0085	+0.0045	-0.0040
40	WD-FN	199	-0.0054	-0.0053	-0.0001
41	WD-PN	271	-0.0014	-0.0023	+0.0009
42	WD-KN	181	+0.0053	+0.0072	+0.0019
43	FN-PN	133	-0.0060	-0.0064	+0.0004
44	FN-OS	209	-0.0055	-0.0032	-0.0023
45	FN-KN	190	+0.0010	+0.0048	+0.0038
46	PN-OS	102	-0.0066	-0.0006	-0.0060
47	PN-KN	151	+0.0037	+0.0058	+0.0021
	Probable error		±0.00402	±0.00355	

new velocity and adjust the shoran network as a whole, holding only one triangulation station and the azimuth to another. Nevertheless by the present criteria the shoran results appear to be excellent.

It is particularly noteworthy that the greatest reductions in the v's occurred in the two sliver triangles TN-PN-OS and PH-OS-NL.

Conclusions--The writer suggests that this approach to the determination of the velocity of propagation of radio waves may furnish a better value than the methods used heretofore. The method of determining velocity by shoran has certain advantages:

(1) The distances are so long that the limited errors of shoran do not cause large proportional errors. Where measurements are made at about the maximum shoran range the derived values of velocity acquire considerably more accuracy.

(2) The indicated precision of first-order triangulation is an excellent standard of comparison.
(3) The internal agreement of the shoran data is remarkably high and indicates an observational precision greater than the published results of the previous light-velocity measurements.

It is particularly important to note that triangulation of a very high order can be executed by shoran. The geodesist has a new tool at his disposal by means of which control can be extended over large unsurveyed areas, inter-continental ties can be made, and isolated island groups connected. During the course of these surveys repeated comparisons of shoran measurements with distances determined by triangulation will be made. In this manner, data will continually be collected to corroborate or modify the value suggested in this paper for the velocity of propagation of radio waves. The importance of this by-product in future physical research should not be underestimated.

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