

THE VELOCITY OF LIGHT

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THE importance of the velocity of light in the development of electrical theory and practice is sometimes overlooked and in this review special emphasis will therefore be given to this aspect of the work and to the determinations of the velocity by electrical methods. It will moreover be assumed at the outset that in accordance with theory the value is independent of the frequency of the waves. The electrical, radio, and optical methods are all measurements of the same constant and the different titles of the various papers are merely indications of the particular technique employed. If there are discrepancies in the results then the theory must be re-examined; but so far no significant discrepancies have been found.

The suggestion that light consists of an electromagnetic radiation was first made rather tentatively by Faraday in 1846, and ten years later Weber and Kohlrausch obtained a value by an electrical method which was in good agreement with Fizeau's optical value. Faraday's suggestion, and this experimental support, were much in Maxwell's mind when he wrote his famous paper on the electromagnetic field. It may be regarded as an experimental fact which stimulated the formulation of electromagnetic theory rather than as a deduction made from it, as is often supposed.

The early electrical methods are described as the determination of the ratio of the electromagnetic to the electrostatic unit, and to see how this arose it is necessary to consider the historical development of electrical units. Coulomb's law of the force between two charges q_1 and q_2 at a distance d in a medium of relative permittivity ϵ is to-day written in the M.K.S. system of units as :

$$F = \frac{q_1 q_2}{4\pi\epsilon\epsilon_0 d^2} \quad . \quad . \quad . \quad . \quad (1)$$

and the analagous law for the force between two magnetic poles p_1 and p_2 in a medium of relative permeability μ as :

$$F = \frac{p_1 p_2}{4\pi\mu\mu_0 d^2} \quad . \quad . \quad . \quad . \quad (2)$$

where the constants ϵ_0 and μ_0 are known as the permittivity and permeability of free space. In the early development of the subject however the force equations were preserved in their simplest possible form by the omission of the constants $4\pi\epsilon_0$ and $4\pi\mu_0$. The simplified equations were used for the definition of charge and magnetic pole strength and two different systems of units, the electrostatic and electromagnetic, were thus obtained. Problems soon arose involving both electric and magnetic effects; the relationship between the units was therefore required, and was determined by measuring the same quantity in both systems. One of the earliest measurements was made by Maxwell himself, and it is interesting to recall his introductory paragraph.

“The importance of this ratio in all cases in which electrostatic and electromagnetic actions are combined is obvious. Such cases occur in the ordinary working of all submarine telegraph cables, in induction coils and in many other artificial arrangements. But a knowledge of this ratio is I think of still greater importance when we consider that the velocity of propagation of electromagnetic disturbances through a dielectric medium depends on this ratio, and according to my calculations is expressed by the very same number” (Maxwell, 1868).

To-day we should regard the measurements as giving the value of $\frac{1}{\mu\mu_0\epsilon\epsilon_0}$ in equations (1), (2), and it is clear that while one of the quantities ϵ_0 or μ_0 can be given an arbitrary value the other must be adjusted to maintain the relationship $c = 1/\sqrt{(\mu_0\epsilon_0)}$. These results are now incorporated in electromagnetic theory in which the velocity of propagation of electromagnetic waves in a uniform dielectric of infinite extent is given by

$$v = c/\sqrt{(\mu\epsilon)} \quad . \quad . \quad . \quad . \quad (3)$$

Most of the measurements have been carried out in air, but the term $\sqrt{(\mu\epsilon)}$ is then only about 3×10^{-4} greater than unity, so that the velocity is readily reduced to the free space value. In the optical experiments it could be assumed that the medium was of infinite extent because, compared with the wave-length of light, the boundaries of the beam were very great and had a negligible effect on the propagation. This condition can be achieved, although with more difficulty, in some of the methods using radio waves, but in the cavity resonator experiments the velocity is strongly affected by the dimensions of the apparatus, and the free space value must be calculated from the measured results by the use of Maxwell's electromagnetic theory. All the methods involve the

measurement of distance and time interval. The time intervals are so small that they can be obtained and measured with the required accuracy only by means of some regular vibrations such as those of a tuning fork or of a piece of quartz. The frequency of vibration and hence the time between successive vibrations is measured by reference to a frequency standard. Use is thus made of a remarkable feature of frequency-measuring techniques, *viz.* that the same proportional accuracy can be achieved in the measurement of a frequency of 10^{10} c/s (or repeated time interval of 10^{-10} s) as in the measurement of a frequency of 10^{-5} c/s (or repeated time interval of one day). The distances involved in free wave propagation are simply those travelled by a pulse of light or radio waves, but in guided wave propagation the dimensions of the guide are also important. In the electrical circuit measurements the concept of a wave travelling a certain distance is lost altogether. The values of an inductance and a capacitance are calculated in terms of their dimensions, ϵ_0 and μ_0 , and measured in terms of the fundamental units of mass length and time.

The methods can therefore conveniently be divided into three groups, based respectively on the lumped electrical circuit, free wave propagation, and guided wave propagation.

ELECTRICAL CIRCUIT METHODS (RATIO OF THE ELECTROMAGNETIC TO THE ELECTROSTATIC UNITS)

More than twenty determinations have been made by these methods. The accuracy achieved in the early attempts was only 1 per cent., but Rosa and Dorsey (1906) by an extremely thorough and painstaking piece of work obtained the most accurate value of the constant known at that time. A number of capacitors in the form of plates, spheres, and cylinders were constructed with the greatest possible precision and their values were calculated from their dimensions. The capacitance can be expressed as :

$$C = \epsilon\epsilon_0 C^1 \quad . \quad . \quad . \quad . \quad (4)$$

where C^1 is a function of the dimensions, and this is equivalent to measuring C in electrostatic units. The capacitance was then measured in terms of a resistance by two methods—a Maxwell's bridge and a differential galvanometer. The result in both cases can be written :

$$C = \frac{R_1}{R_2 R_3 f_1} = \frac{1}{R} \frac{A}{f_1} \quad . \quad . \quad . \quad . \quad (5)$$

where R_1 , R_2 , R_3 are resistors expressed in international ohms, R is the value of the absolute unit of resistance in ohms, A is the

quantity determined by the experiment and f_1 is the frequency at which the capacitor is charged and discharged. The next step was to determine the international ohm in electromagnetic absolute units. In common with others who used this method, Rosa and Dorsey did not make this measurement themselves, but used the internationally accepted value obtained at the N.P.L. and other national standardising laboratories. One method is that due to Lorenz, in which a metal disc is rotated in a magnetic field and the e.m.f. between the rim and centre of the disc is balanced against that across a resistor. The field is produced between a system of two coils of calculable mutual inductance with respect to the disc and carrying the same current as that in the resistor. The value of the resistor can be expressed as :

$$R = \mu\mu_0 f_2 M \quad . \quad . \quad . \quad . \quad (6)$$

M being Newmann's Integral, which is a function of the dimensions of the coils, and f_2 the frequency of rotation of the wheel. From (4), (5), and (6) we have :

$$C = \epsilon\epsilon_0 C^1 = \frac{A}{f_1} \frac{1}{\mu\mu_0 f_2 M}$$

or

$$\mu\mu_0 \epsilon\epsilon_0 = \frac{A}{C^1 M f_1 f_2} = \frac{1}{v^2} \quad . \quad . \quad . \quad . \quad (7)$$

The total spread of a very large number of observations was ± 150 km/s, but Rosa and Dorsey estimated that the maximum uncertainty including systematic errors was not more than ± 30 km/s. To this must be added the error in the value of the absolute unit of resistance which is now only ± 1 part in 10^5 or 3 km/s in the velocity of light, although it was much greater at the time of the experiments. The observational errors could no doubt be reduced if the experiment were repeated with modern techniques, but the difficulties of constructing the capacitors and inductors and of calculating their values remains and it would not be easy to effect a worth-while improvement in the overall accuracy.

FREE WAVE METHODS

The astronomical measurements of Roemer and Bradley are early examples of free wave methods, but they cannot furnish an accurate value of c because of the large observational errors and the uncertainties in the distances involved. Galileo is reported to have made the first attempt to measure the time of travel of a wave of light over a measured distance on the earth's surface. He was right in his surmise that light has a finite velocity and his

method was sound, but he naturally failed to obtain a result with such crude apparatus as lanterns fitted with manually operated shutters. Even with modern techniques it would be difficult to measure the time of a single transit, and the experiments are therefore devised so that a regular succession of waves or of pulses of waves produces a stationary effect which can be observed at leisure. A well-known method of obtaining such a stationary effect is to allow two beams travelling in opposite directions to interfere. If a beam is reflected back along the same path the amplitudes of the waves at a time t can be represented by :

$$E \sin 2\pi\left(ft - \frac{x}{\lambda}\right), E \sin 2\pi\left(ft + \frac{x}{\lambda} + \phi\right)$$

where E is the intensity, f the frequency, λ the wave-length, x the distance measured from the point of reflection ; and $2\pi\phi$ the change of phase on reflection. The resultant of the two trains of waves is given by :

$$2E \cos 2\pi\left(\frac{x}{\lambda} + \frac{\phi}{2}\right) \times \sin 2\pi\left(ft + \frac{\phi}{2}\right)$$

which represents a system of standing waves, the amplitude being zero at distances differing by $\lambda/2$. The wave-length can be obtained by observation of the points of zero intensity and the velocity of the wave is then simply :

$$v = f\lambda$$

Alternatively the distance d or frequency can be adjusted to give a whole number n of half wave-lengths in the path when :

$$v = 2df/n$$

Standing wave methods are commonly used for measuring the velocity of sound and can, as we shall see later, be used for radio waves, but they cannot be used in the case of light waves for two very good reasons. In the first place the waves from a light source are incoherent, *i.e.* they consist of short trains of random phase which would destroy the standing wave pattern. This difficulty can be overcome by dividing the beam as in a Michelson interferometer and then combining the two parts after they have travelled along two paths of nearly equal lengths. They interfere with the production of fringes in the optical detecting systems and if one path is changed by $\lambda/2$ the pattern moves across the field of vision by one fringe. In this way the wave-length of light can be measured with a precision of 2 parts in 10^8 under the best laboratory conditions, but the second difficulty—that of measuring the frequency of light waves—has not yet been overcome.

The beam is therefore modulated in intensity or chopped into sharp pulses by such means as a rotating toothed wheel, a rotating mirror, a Kerr cell, or a vibrating quartz crystal, and the modulated beam, after reflection at a distant mirror, is returned to a detector the sensitivity of which is varied at the modulation frequency. If the time of transit equals or is a multiple of that between successive peaks of intensity, the detector is in its most sensitive condition when the light peak returns and a maximum signal is obtained. As the frequency of modulation is increased the detected signal will pass through alternate maxima and minima of intensity. Either the frequency or the distance is adjusted to give a minimum signal and the velocity is :

$$v = 2df/(2n - 1)$$

where n is the number of peaks in the total path d . Taking into account the effect of the atmosphere the free space value becomes :

$$c = 2df\sqrt{(\mu\epsilon)}/(2n - 1)$$

An additional small correction must be applied because air is dispersive at optical frequencies and the group velocity, which is the quantity measured, differs slightly from the free wave phase velocity. The main difficulty of the experiment is that of judging the minimum intensity and, since the accuracy of this setting can be expressed as a fraction of the distance between intensity peaks, it is advantageous to increase the distance or frequency to give as many peaks in the path as possible. Frequencies between 20 kc/s and 100 Mc/s have been used, but unfortunately, except for the recent experiments of Bergstrand, the higher frequencies were used with shorter path lengths and there was no gain in accuracy, the total spread of the observations being in general about 100 km/s. The most famous of the optical experiments is that due to Michelson, Pease, and Pearson (1935), the feature that caught the imagination being the evacuated mile-long pipe in which the beam of light was reflected backwards and forwards to give a total path length of 10 miles. The repetition frequency of the pulses of light produced by a rotating mirror was only 20 kc/s and the setting accuracy was not higher than in other similar experiments. The results indicated moreover that there were quite large unresolved systematic errors, and in the circumstances it seems doubtful whether the use of an evacuated pipe was justified. The total effect of the air on velocity is about 85 km/s and if temperature, pressure, and humidity are measured at a number of points in the path it should be possible to calculate the correction to ± 1 km/s.

The value obtained from this experiment was $299,774 \pm 11$ km/s and, as several subsequent determinations using very different optical systems gave results in close agreement, a value of $299,776 \pm 4$ km/s was adopted by Birge (1941) with some confidence.

A notable advance in optical methods has recently been made by Bergstrand (1950), who used a fairly high frequency (8 Mc/s) together with a long path. He also improved the observational accuracy by using two trains of waves 180° out of phase, following the same path but spaced in time by 0.01 s. They were produced by applying to the light source a 50 c/s square wave modulation in addition to the high frequency modulation. The received signals were detected separately by an instrument of relatively long period and balanced in opposition so that the setting could be made to a sharp zero of intensity instead of to a flat minimum. The spread of the observations was reduced to a few km/s and the result was in close agreement with that obtained by the cavity resonator method described later.

Another free wave method, based on radar, is perhaps the simplest in principle of all the methods of measurement. The time of travel of a pulse of radio waves to a distant target and back again is recorded directly on a time scale derived from a frequency standard. In the practical application of radar, Birge's value of velocity was assumed and the frequency of the standard was chosen so that the time scale gave a direct reading of distance, sharp pulses being obtained at time intervals corresponding to target distances of 10, 1, and 0.1 miles. The timing pulses produced vertical deflections of the trace on a cathode ray tube, the horizontal speed of the trace being sufficient to give adequate separation. Both the radar pulses and the horizontal movement of the time base of the cathode ray tube are repeated at regular intervals synchronised with the frequency standard, so that the time markers appear to remain stationary, while the radar pulse returned from the target moves steadily along the scale as the distance changes. Fig. 1 is reproduced from a photograph obtained by Jones and Cornford (1949), in which the small deflections are the mile time marks and the large deflection is caused by a radar pulse returned from a target 31 miles away. In order to obtain the necessary reading accuracy only a small part of the scale is displayed at the same time, but the number of mile marks occurring since the transmission of the pulse is counted electrically and indicated by the pointers on the figured scale in the top right-hand corner of the picture.

The radio frequency of the pulses does not enter directly into

the measurements but it must be fairly high to give a sharp, clearly defined reading on the time scale and also to obtain conditions of free wave propagation. It is known that a low frequency wave travels over the ground with a velocity considerably less than the free wave value, and the path should therefore be an optical one at least several wave-lengths above the ground. Smith, Franklin, and Whiting (1947) used the "GH" system with a frequency of about 50 Mc/s and a path between two ground stations, Jones and Cornford the "Oboe" system with a frequency of 3000 Mc/s and a path between a plane and two ground stations, and Aslakson (1949) the "Shoran" system with frequencies around 300 Mc/s and a similar but longer path. In all of these systems the pulse is not simply reflected, but is retransmitted by a responder, and

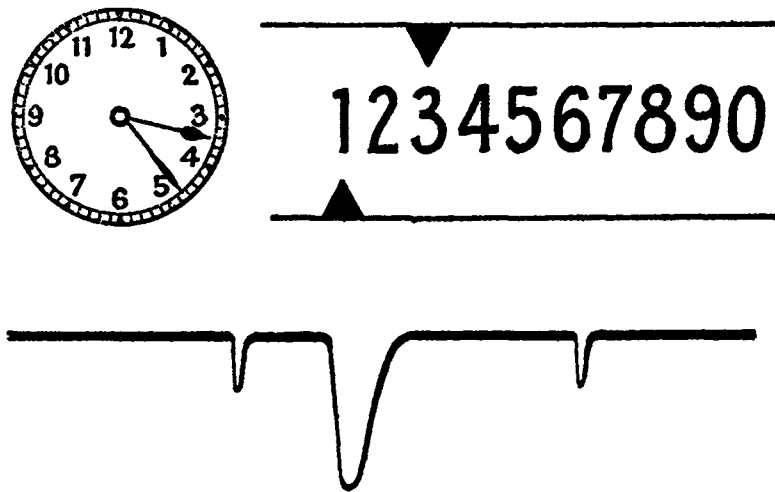


FIG. 1.—Radar display.

the sensitivity and range are thereby greatly increased. The time delays in this and in other parts of the equipment can be measured directly and allowed for in the calculation of distance.

In order to obtain a direct measure of the velocity of radio waves and to investigate the possibilities of the technique for geodetic surveying, comparisons have been made between distances obtained by both radar and normal triangulation methods. The scheme used by Jones and Cornford is shown in Fig. 2. The distance measured is that between the two ground stations A and B. For the radar measurements a plane flies at constant height across the line joining the two stations and its distance from each is measured. The distances are a minimum when the plane is exactly in line and from these minimum distances and the height of the plane the distance AB is calculated. Aslakson's method differed slightly in that the plane was between the two stations when it

crossed the line joining them and the minimum sum of the distances was obtained when the plane was vertically above the line. In both of these techniques the signal on the cathode ray tube time scale will move to a minimum and then reverse its direction. Aslakson found that a higher reading accuracy was obtained if the plane crossed the line at an angle so that the signal moved steadily along the scale. He therefore adopted a "figure-of-eight" technique, each measurement being obtained from four crossings made at an angle of $12\frac{1}{2}^\circ$ to the perpendicular between the stations. In his measurements a plane followed the path of the pulse in

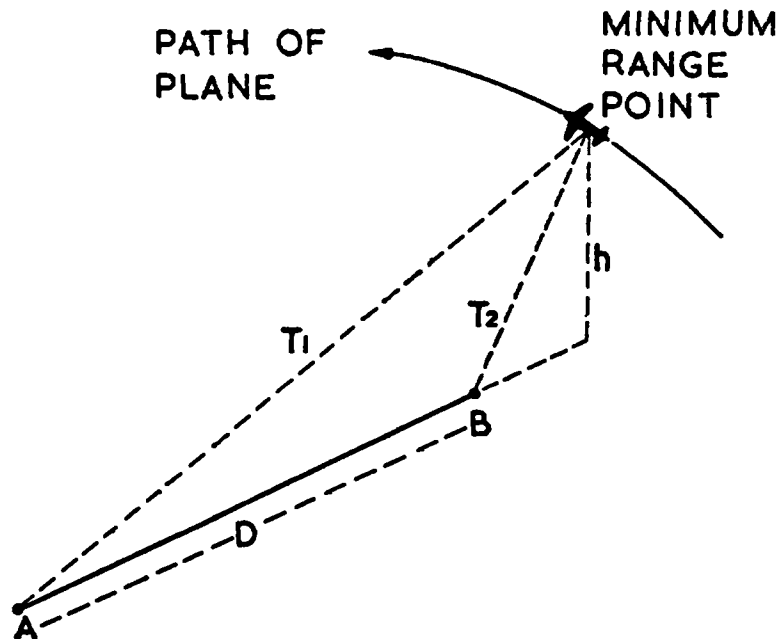


FIG. 2.—The measurement of velocity by radar.

D is a known distance.

h is the height of the plane.

v is determined from D , h and the times of travel T_1 , T_2 of the radar pulses.

order to obtain precise measurements of the atmospheric conditions and to enable an accurate correction to be applied instead of the average correction used in the calculation of the radar tables. By measuring a whole system of base lines Aslakson was able to claim an accuracy of ± 2.4 km/s for his value of velocity.

The final free wave method to be discussed employs a new and potentially important technique which is however still in course of development. Very short radio waves of 3 cm. or less bear obvious similarities to light waves, and they can be used to demonstrate in the laboratory the properties of reflection, refraction, and diffraction. Experiments with a radio wave Michelson interferometer were made in Germany during World War II, and it was decided at the National Physical Laboratory to explore its possi-

bilities for the measurement of lengths greater than can be dealt with by optical interferometry. Initial work on the project was carried out by Culshaw (1950) at T.R.E. and it is now being continued at the N.P.L. It differs from the optical interferometer in two fundamental respects. The source of radiation, being a continuous wave oscillator accurately controlled in frequency, is coherent, and the length of path over which interference phenomena can be observed is limited only by considerations of the intensity of the signal. Moreover, whereas in optical interferometry the wave-length of a carefully defined source of light constitutes the standard of length, this is not true with radio waves. The wave-length is calculated from the frequency of the source and the velocity and if the scheme is to be practicable for length measurement a precise knowledge of the velocity is required. Before optical interferometry could be used for length measurements it was necessary to measure the wave-length of the light in terms of the standard metre; and before radio interferometry can be used it will be necessary to measure velocity in terms of the metre and the unit of time. When this has been done subsequent measurements can be made in terms of the frequency of the source, which can be determined with great accuracy by standard techniques. Radio wave interferometry does however present special difficulties because the wave-length is by no means negligibly small in comparison with the mirrors and the distance from surrounding objects. Diffraction effects become important and corrections must be determined by experimental and theoretical investigations.

A point to notice in connection with the radar and interferometer methods is that the change in velocity caused by the atmosphere is more difficult to assess at radio than at light frequencies, because the refractive index of water vapour is then much greater than that of dry air, instead of being nearly the same. The exact amount of water vapour present must therefore be carefully measured. The actual values of the refractive indices of air and water vapour were not known with sufficient accuracy for interferometry work and they have therefore been measured at the frequency of operation (24,000 Mc/s) (Essen and Froome, 1951). The results suggest that the value adopted by Aslakson in his calculations was slightly too low, whereas that used in the British radar experiments was considerably too high, causing an error of 5 km/s in velocity. In the table of results given later, Essen and Froome's value has therefore been used to derive the velocities *in vacuo* from the results of Smith, Franklin and Whiting, Jones, and Jones and Cornford.

GUIDED WAVE METHODS

The measurement of the velocity of waves along conductors dates back to 1891, when Blondlot coupled a pair of parallel wires to an oscillator and altered their effective length by sliding a short circuit reflector along them. At successive positions differing by $\lambda/2$ the system resonated and reacted on the oscillator; and in this way he measured wave-lengths of 8 m and 35 m and found that the velocity, between 292,000 and 303,000 km/s, was substantially the same as that of light. Similar measurements were made by Trowbridge and Duane, and Gutton in 1911 measured the effect of the dimensions of the conductors on velocity to check the theoretical work of Mie and Sommerfeld. All these measurements were made under difficult experimental conditions because the only oscillators available produced trains of damped waves; but Mercier (1923) made a precise determination of velocity by this method, using continuous waves at frequencies between 46 Mc/s and 66 Mc/s. The parallel conductors, usually known as Lecher wires, were 11 m long and tightly stretched between the walls of the room at a distance of 2 m from the floor and neighbouring objects. The movement of the short circuit to give successive resonant positions was measured by an invar tape which could be read to an accuracy of 0.1 mm. The velocity obtained was 299,575 km/s, and after corrections had been applied for the effect of the conductors and the atmosphere this gave a free space value of 299,782 km/s \pm 30 km/s. It was a remarkably careful piece of experimental work which seems however to have attracted little attention.

The recent methods using cavity resonators are similar in principle, but the wave is guided by a hollow metal tube in place of the parallel conductors. A wave travels down a cylindrical tube with little attenuation if the frequency exceeds:

$$\frac{1}{2\pi} \frac{c}{\sqrt{(\mu\epsilon)}} \frac{r_{1m}}{a}$$

where r_{1m} is a root of the Bessel Function J_1 and a is the internal radius of the cylinder. The propagation is affected by the walls and the phase wave-length λ_g and velocity are greater than those in free space. The phase velocity is given by:

$$v_p = f\lambda_g = \frac{2\pi f}{\sqrt{\left(\frac{4\pi^2 f^2 \mu\epsilon}{c^2} - \left(\frac{r_{1m}}{a}\right)^2\right)}}$$

As a is increased the phase velocity naturally approaches c , but

with convenient dimensions at centimetric wave-lengths it may be as much as 30 per cent. greater than c . It is however clear that if f , λ_g and a are measured c can be calculated. The formula applies for a perfectly conducting tube, but in practice for imperfectly conducting materials the fields penetrate a small distance into the walls and the effective diameter is increased. This effect can be calculated and an appropriate correction applied. The easiest way of measuring λ_g is to launch a wave through a tiny aerial or coupling hole in the cylinder closed by reflectors at both ends and to adjust the frequency of oscillation or the length of the cylinder until the length contains a whole number of half-wave-lengths. The waves reflected at the ends are then all in phase

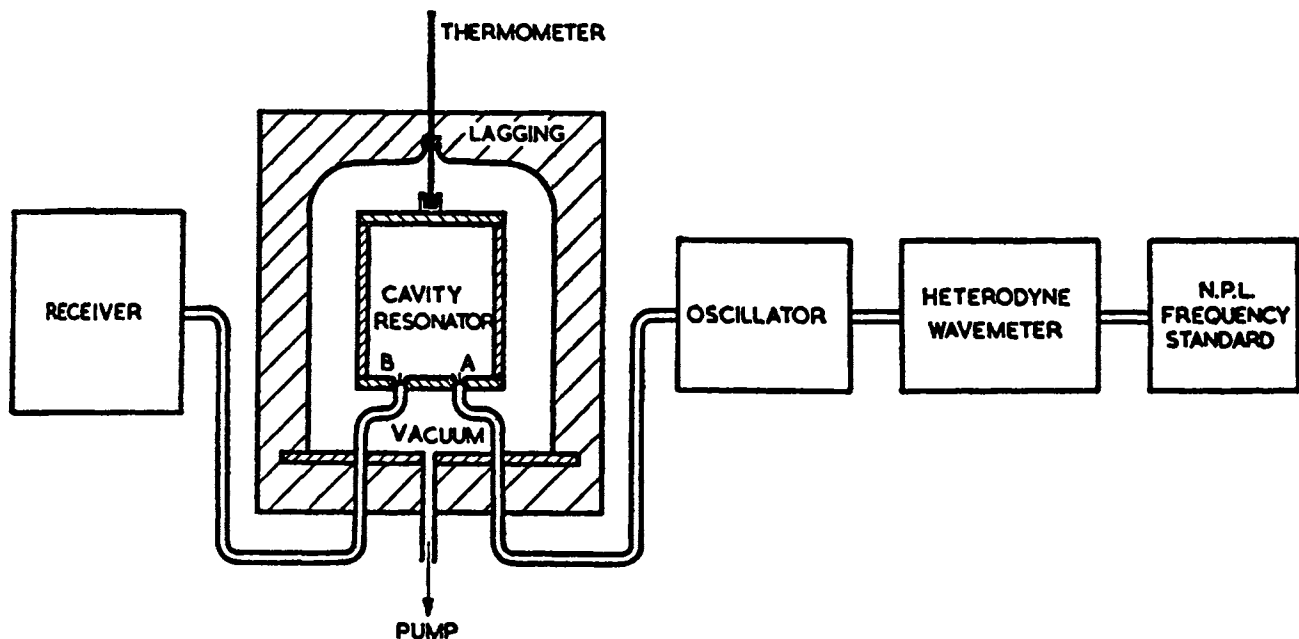


FIG. 3.—Schematic representation of the cavity resonator experiment. The velocity is determined from the resonant frequency and the dimensions of the resonator.

and there is a large increase in the field detected by another similar aerial. This resonance effect is very sharp and the resonant frequency can be set and measured with a precision exceeding 1 part in 10^6 . The experimental arrangements using a fixed cavity resonator in the measurements at the N.P.L. (Essen and Gordon-Smith, 1948), which first cast doubt on the accepted optical value of c , are shown in Fig. 3, and the cavity in more recent work there (Essen, 1950) is shown in Fig. 4. The length of the resonator in Fig. 4 is adjusted by movement of the plunger to successive positions of resonance and λ_g is thus obtained from the difference between these positions which are determined by means of a block of precision gauges between the piston and a base plate. In practice it is more convenient, especially as the cavity is contained in an

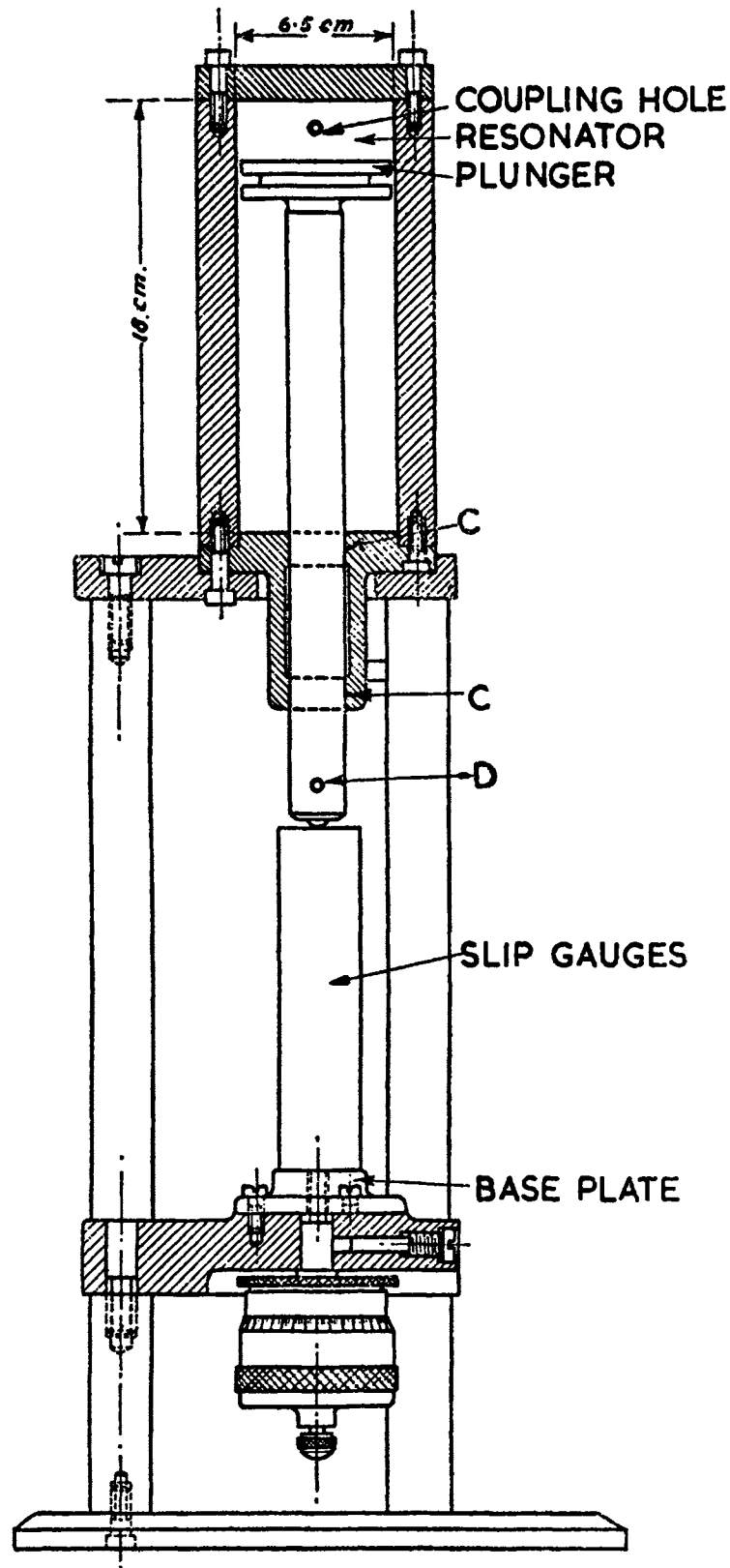


FIG. 4.

evacuated enclosure, to set the plunger at the calculated positions and to adjust for resonance by varying the frequency. The variation needed is only a few parts in 10^5 and it is easy to convert

the results to the required form of a series of resonant lengths corresponding to a fixed frequency. The principal uncertainties are the value of the diameter and the effect of small imperfections of the cylindrical wall; but by using two different modes of resonance it is possible to measure the effective diameter in terms of frequency, which can be measured with a precision of better than 1 part in 10^6 , and the length of a set of gauges, which can be checked by interferometer measurements with the same precision, and since, moreover, λ_g is determined by differences, certain small end effects are eliminated and the effects of mechanical and electrical imperfections of the walls and ends of the cylinder are greatly reduced.

A cavity resonator method has also been used at Stanford University (Bol, 1950) and a provisional result, together with a general description of the work, have been released. The main difference from the N.P.L. method is that the effects of the coupling and of other geometrical imperfections are calculated rather than eliminated by the experimental technique. The effect of surface imperfections cannot be eliminated experimentally or calculated accurately, but it will be less serious than in the N.P.L. work because the resonator is larger and the frequency lower.

TABLE I

VELOCITY OF LIGHT DETERMINATIONS

(a) *From the Ratio between the Electromagnetic and Electrostatic Units*

Date.	Author.	Velocity in vacuo (km/s).*	Probable error (\pm km/s).
1857	Weber and Kohlrausch	310,800	
1868	Maxwell	284,300	
1869	Thomson, W., and King	280,900	
1874	McKichan	289,700	
1879	Ayrton and Perry	296,100	
1880	Shida	295,600	
1883	Thomson, J. J.	296,400	
1884	Klemencic	302,000	
1888	Himstedt	300,660	
1889	Thomson, W.	300,500	
1889	Rosa	300,090	200
1890	Thomson, J. J., and Searle	299,690	
1891	Pellat	301,010	
1892	Abraham	299,220	
1897	Hurmuzescu	300,190	
1898	Perot and Fabry	299,870	
1899	Lodge and Glazebrook	301,000	
1906	Rosa and Dorsey	299,784	30

* In some cases the published result has been adjusted later by the experimenter, de Bray, Birge or the author.

TABLE I (continued)
(b) By Free-Wave Methods

Date.	Author.	Approx. Distance (metres).	Approx. Frequency (Mc/s).	Method Used.	Velocity in <i>vacuo</i> (km/s).*	Limits of error (\pm km/s).
1676	Roemer	3×10^{11}		astronomical	300,000	
1849	Fizeau	9,000	0.009	TW LP	315,300	500
1862	Foucault	20		RM DL	298,000	500
1874	Cornu	23,000	0.05	TW LP	300,400	200
1879	Michelson	700		RM DL	299,910	50
1882	Newcomb	3,700		RM DL	299,860	30
1882	Michelson			RM DL	299,850	60
1902	Perrotin	46,000		TW LP	299,880	80
1924	Michelson	35,000	0.004	RM LP	299,802	30
1926	Michelson	35,000	0.004	RM LP	299,798	4
1928	Karolus and Mittelstadt	200	5	KC LW	299,786	20
1935	Michelson, Pease and Pearson	1,600	0.02	RM LW	299,774	11
1937	Anderson	170	19	KC LW	299,771	15
1940	Hüttel	80	10	KC LW	299,771	10
1941	Anderson	170	19	KC LW	299,776	14
1947	Smith, Franklin and Whiting	130,000		R RP	299,786	50
1947	Jones	70,000		R RP	299,782	25
1949	Bergstrand	9,000	8	KC LW	299,793	2
1949	Aslakson	300,000		R RP	299,792	2.4
1949	Jones and Cornford	150,000		R RP	299,783	25
1950	Bergstrand	7,000	8	KC LW	299,793.1	0.25
†1950	Houstoun	78	100	VQ LW	299,775	9
†1950	McKinley	20	8	VQ LW	299,780	70

TW = toothed wheel
RM = rotating mirror
KC = Kerr cell

R = radar
VQ = vibrating quartz
LP = light pulse

DL = deviation of light beam
LW = light wave
RP = radio pulse

* In some cases the published result has been adjusted later by the experimenter, de Bray, Birge or the author.

† These results are regarded as indicating the potentialities of the vibrating quartz method rather than as precise determinations.

(c) By Guided Wave Methods

Date.	Author.	Approx. Frequency (Mc/s).	Method.	Velocity (km/s).	Probable Error (\pm km/s).
1891	Blondlot	10	Lecher wires	295,000 to 305,200	
	Trowbridge and Duane	5	"	292,000 to 303,600	
1923	Mercier	75	"	299,782	30
1947	Essen and Gordon-Smith	3,000	Cavity resonator	299,792	3
1950	Essen	10,000	"	299,792.5	1
1950	Bol	3,000	"	299,789.3	0.4

RESULTS

It may be of historic interest to conclude this review with a fairly complete table of results, although in view of their high precision the values obtained recently by Bergstrand and Aslakson and by the cavity resonator methods must have much greater weight than the others in any assessment of the most probable value. Beardon and Watts (1951), Essen (1951) and Stille (1951), using the evidence in different ways, all give this value as :

$$c = 299,790 \text{ km/s}$$

The limits given in the last column of the table of results are called a probable error, but not in the strict mathematical sense. The assessment of errors is always a difficult matter. When the spread of the individual observations is large, as in the majority of these results, the random errors can be reduced by taking a large number of observations, but in these circumstances it is impossible to make any experimental check of small systematic errors. In the most precise measurements with cavity resonators the spread is less than ± 0.5 km/s and the random error could be made still smaller by taking the average of a number of observations. But there is little point in doing this until the systematic errors can also be reduced.

It is often said that the experimenter is the last person to assess the accuracy of his results, and it must be admitted that he frequently treats this part of his work too lightly. He is however in a better position to judge than the reviewer, and it is not altogether surprising that, even after their careful and detailed studies, both Birge and Dorsey arrived at a result which now appears to be considerably too low. Although Birge is careful to point out that the limits he gives are relative rather than absolute, great weight was attached to his final figure of $299,776 \pm 4$ km/s, which indeed appears to have given the radio engineer an altogether false impression of the accuracy of the optical results. It can only hinder the progress in a subject to make the limits closer than is justified.

It would be useful if the experimental limits of accuracy were expressed in three parts, the standard deviation of the individual observations, the systematic error due to known causes, and an estimate of other possible systematic errors due to causes not fully understood. Presented in this way the great advance made in the velocity determinations during recent years would stand out more clearly. The precision has increased to such an extent that the equipments employed can be developed into measuring instruments,

used for determining length in terms of velocity and frequency. The velocity of light has therefore become a standard of measurement having applications in the field of metrology as well as those of electrical and radio engineering.

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A full account of the early measurements is given in the reports of the Congrès International de Physique, **2**, 1900.