THE CALIBRATION OF AERIAL SURVEY CAMERAS

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Abstract

Two experimental procedures for aerial survey camera calibration using respectively a Wild T4 theodolite and a modified Cooke, Troughton and Simms geodetic theodolite are described. The procedures determine radial distortion only; tangential distortion is assumed to be negligible. Details of a graphical method of computing the results are given in an appendix.

Introduction

Up to the time these experiments were initiated the Ordnance Survey had calibrated aerial survey cameras at a nearby government research establishment. The equipment used had been specially constructed for the establishment's own purposes, and gave results of a high standard of accuracy. These calibrations are called "Standard" in this paper. The impending move of the Ordnance Survey to Southampton in 1968 made it imperative that some alternative facilities for calibration be found to replace the existing local arrangements, and preferably the Ordnance Survey should have its own facilities.

Several possibilities were considered including custom-built equipment on the lines of those constructed in other establishments, commercially available equipment, and methods involving the adaptation of available survey instruments. The Ordnance Survey makes about twenty calibrations a year, and this limited requirement was related to the estimated financial outlay on the various methods considered. This indicated that the most promising methods were one using a Wild T4 theodolite, and another in which a Cooke, Troughton and Simms (C.T.S.) geodetic theodolite was adapted to serve as a goniometer. This paper describes the experimental work on these two methods and the results obtained. All prototype equipment was made in the Ordnance Survey workshops.

Calibration Using a Wild T4 Theodolite

Principle. The zenith distance to each of a line of targets in the focal plane of the camera is measured through the camera lens with the camera axis approximately vertical. The camera is set up so that the plane defined by the entrance pupil and the line of targets on the diagonal is vertical. The T4, placed below the camera, is moved to a series of positions where the zenith distances of the targets, seen through the lens, can be measured under the following conditions:

(a) the trunnion axis is normal to the vertical plane through the targets;
(b) the optical axis of the telescope bisects the entrance pupil of the camera.

Experimental procedure.

(1) A Wild 6-inch camera, No. 687, was supported and levelled on a gantry made of 2 inch dural tubing.

(2) A fluorescent strip light was placed above the camera to illuminate the targets seen through the lens, the targets being the réseau crosses on a register glass. (The réseau is centimetric, and the crosses are arbitrarily numbered by row and column with the centre cross 2020; that is, row 20 and column 20.)

(3) The T4 was set up and levelled on a smooth steel bench placed immediately below the camera.

(4) A steel straight-edge was clamped to the surface of the bench to serve as a guide rail for sliding the T4 along a straight line when traversing the line of targets. The positions of the guide rail and T4 were adjusted so that:
   (a) the rail was parallel to the perpendicular plane passing through the line of targets and the entrance pupil;
   (b) the trunnion axis of the T4 was normal to this plane;
   (c) two of the T4 footscrews were kept in contact with the edge of the guide rail to maintain the alignment of the theodolite when the line of targets was traversed;
   (d) the entrance pupil of the camera lay in the plane generated by rotating the optical axis of the telescope about the trunnion axis.

   The sighting device described later for the goniometer method of calibration provides a convenient means of setting the T4 to meet these requirements.

(5) The line of targets was observed by sliding the T4 along the rail to successive positions in which the targets could be bisected with the cross hairs in the telescope, whilst keeping the optical axis collinear with the centre of the entrance pupil of the camera.

(6) Four lines of targets, the two diagonals and the two perpendiculars, were observed in this way. The complete observation of a line of targets comprised a forward and back traverse of all targets by each of two observers. This gave four zenith distances to each target, and the mean of the four was accepted. For the experiment the vertical circle level was levelled exactly for each reading; no attempt was made to read the bubble scale while off-level as this would have required a correction to the zenith reading.

(7) Two operators were employed on the task as follows:
   Operator A positioned the T4 for observing, bisected the target, and booked the readings.
   Operator B levelled the suspension level and the vertical circle level, and read the vertical arc.

Calibration Using a Goniometer

Principle. The horizontal direction to each of a line of targets is measured through the camera lens with the camera axis approximately horizontal. The camera is set up so that the plane defined by the entrance pupil and the line of targets along a diagonal is horizontal. The goniometer is placed under the camera with its axis of rotation perpendicular and bisecting the entrance pupil of the camera, and with the optical axis of the telescope coincident with the horizontal plane through the targets.

Description of prototype equipment. The general arrangement is shown in the photographs (Figs. 1, 2, 3 and 4). These include the various Ordnance Survey cameras (Wild, Zeiss and Williamson) set up in position for calibration. It should be noted that each make of camera has its own cage to hold it in position. The Williamson cage is designed to hold both 12-inch and 6-inch lens Williamson cameras (Fig. 4). The design of each cage is similar so that the following description applies to all the various cage and goniometer assemblies.

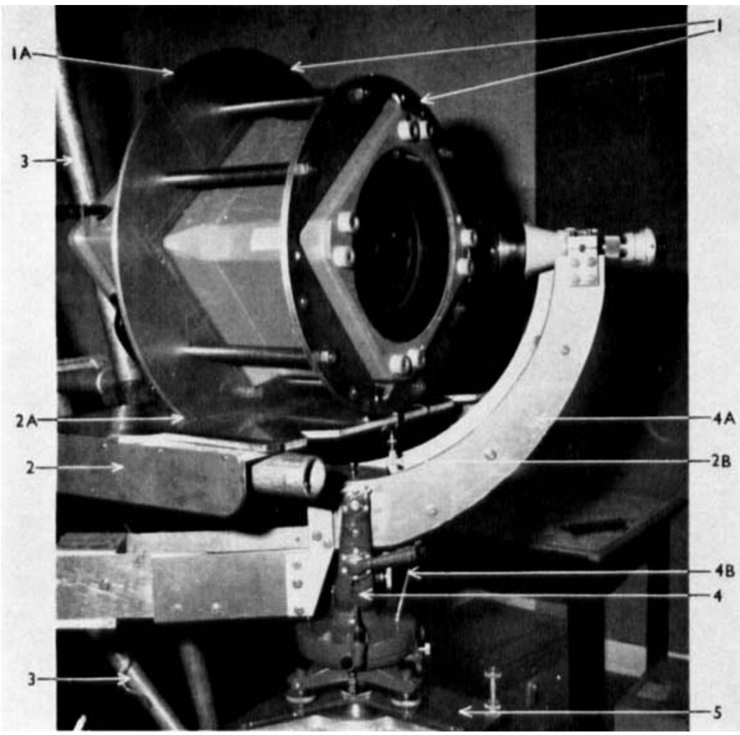


FIG. 1. Wild assembly with goniometer (front view).
(1) Cage holding camera. (1A) Rear ring of cage. (2) Base plate. (2A) Slot in base plate. (2B) Adjustment screw for levelling cage. (3) Scaffolding. (4) Goniometer. (4A) Goniometer arm. (4B) Adjustment screw for levelling telescope. (5) Stand.

The Ordnance Survey Wild (Figs. 1 and 2) and Williamson cameras each carry a register glass with a réseau in the focal plane. The réseau crosses along the diagonals and perpendiculars can be used as targets for calibration. The Ordnance Survey Zeiss 12-inch cameras (Fig. 3) have no register glass, therefore to calibrate these cameras a register glass with a 1-cm. réseau is placed in the focal plane of each camera, the register glass being held in position by means of a frame bolted to the camera.

The ringed cage (1) (see Figs. 1–4) holding the camera is mounted on a base plate (2) containing a slot (2A) which holds the rear ring (1A) of the cage and permits the camera to be rotated about its optical axis. An adjustment screw (2B) on the base plate enables the camera axis to be levelled. A frame of 2-inch dural tubing (3) provides a horizontal rail support (3A) for the base plate. The goniometer (4) comprises a C.T.S. geodetic theodolite with the telescope mounted on an arm (4A) attached to the trunnion axis; it is used to measure the directions of the targets observed through the camera lens. The goniometer has been assembled with the optical axis of the telescope in alignment with the axis of rotation of the arm. The adjustment screw (4B) enables these two axes to be made mutually perpendicular. The goniometer is mounted on a stand (5) provided with three

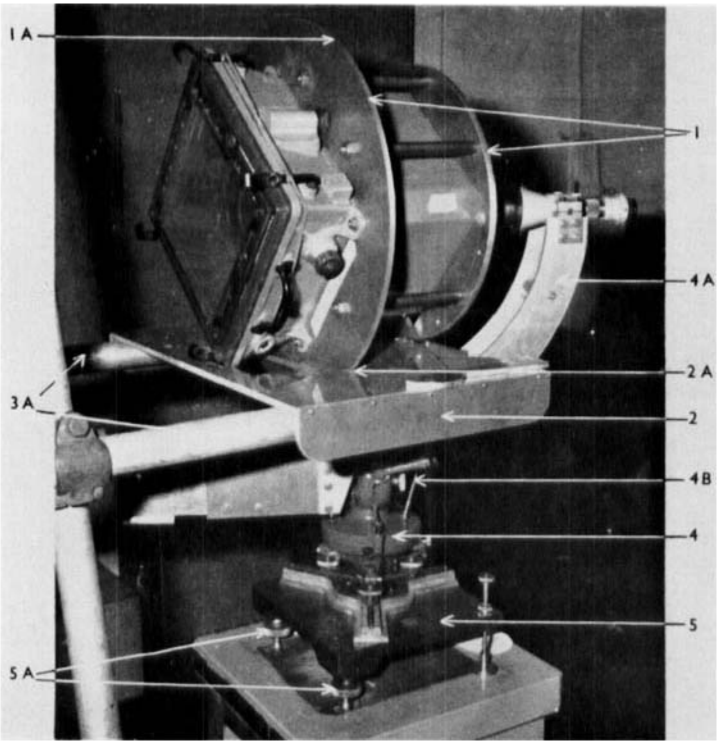


FIG. 2. Wild assembly with goniometer (rear view).
(1) Cage holding camera. (1A) Rear ring of cage. (2) Base plate. (2A) Slot in base plate. (3A) Rails supporting base plate. (4) Goniometer. (4A) Goniometer arm. (4B) Adjustment screw for levelling telescope. (5) Stand. (5A) Screws for raising and lowering goniometer.

adjustment screws (5A) with which to raise or lower the goniometer. A sighting card (6) is used to bring the axis of rotation of the goniometer into coincidence with the entrance pupil of the camera and to make the optical axis of the telescope bisect the entrance pupil. On the card is a circle of the same diameter as the telescope tube with a slot cut across the diameter of the circle. The telescope is pointed at the entrance pupil and the card held in front of the telescope. The slot then acts as a shutter which prevents all light from the entrance pupil entering the telescope except for a narrow beam which is parallel to the optical axis of the telescope. When the telescope is correctly aligned the beam will coincide with the optical axis of the telescope and this will be confirmed by the concentricity of the card circle and telescope tube. Lack of registration of circle and tube will indicate the amount by which the telescope must be moved with respect to the camera and in a direction normal to the direction of the slot to make the beam coincide with the optical axis. A fluorescent strip light (7) placed behind the camera illuminates the targets for observing.

Experimental procedure.

(1) The apparatus was set up in the approximate position for calibration.

(2) The goniometer was levelled by means of the footscrews.

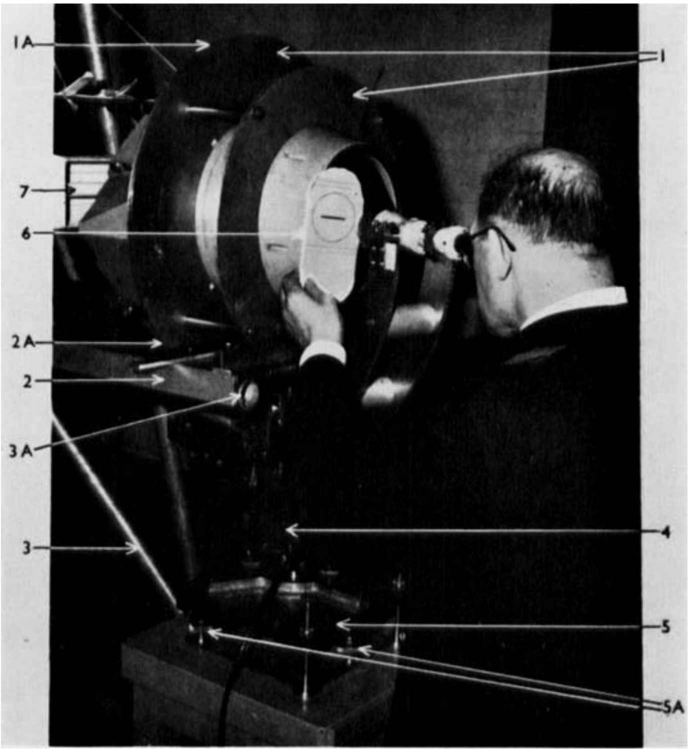


FIG. 3. Zeiss assembly with goniometer.
(1) Cage holding camera. (1A) Rear ring of cage. (2) Base plate. (2A) Slot in base plate. (3) Scaffolding. (3A) Rails supporting base plate. (4) Goniometer. (5) Stand. (5A) Screws for raising and lowering goniometer. (6) Sighting card. (7) Fluorescent strip light.

(3) The centre target was bisected with the cross-hairs of the telescope using the horizontal slow-motion screw and the adjustment screw (2B). This approximately levels the camera axis.

(4) The camera was rotated on its mount about the camera axis until the line of targets could be scanned by swinging the telescope. Should the line of targets when scanned appear to lie on a curve, then, neglecting differential distortion, this will indicate that the axis of the telescope is not horizontal, i.e. it is not normal to the vertical axis of the goniometer and will generate a cone when rotated around this axis. The projection of this cone on the focal plane of the camera will be a curve, hence the apparent curve in the line of targets.

(5) Adjustments were made to the screw (4B) until the line of targets when scanned appeared to be straight. The locking nut (4B) was then tightened. On completion of this adjustment it should not be necessary to readjust this setting for subsequent calibrations unless the goniometer is dismantled.

(6) The sighting card was used as follows to position the entrance pupil of the camera over the goniometer axis:

(a) With the camera set at minimum aperture the telescope axis was positioned approximately parallel to the camera axis and the card

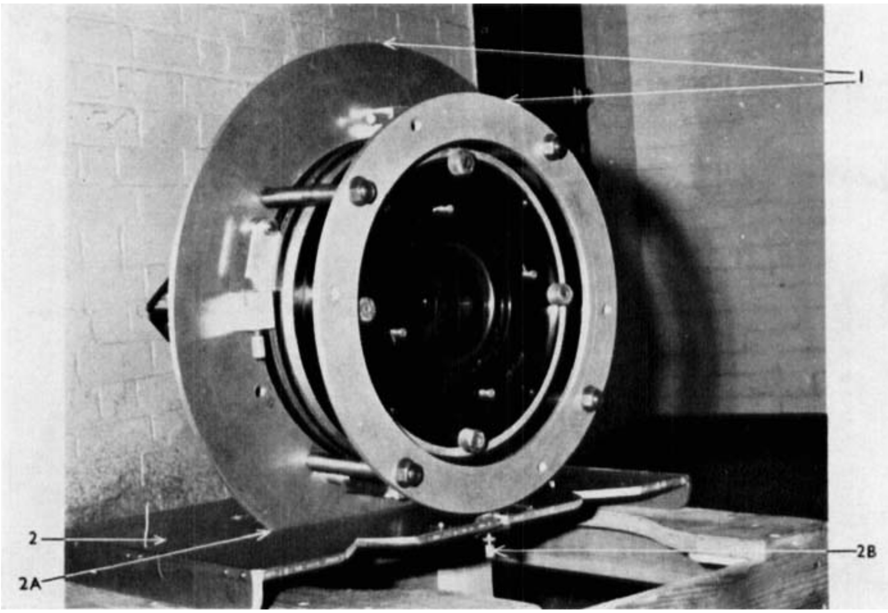


FIG. 4. Williamson F49 assembly.
(1) Cage holding camera. (2) Base plate. (2A) Slot in base plate. (2B) Adjustment screw.

placed in front of the telescope with its slot horizontal. The card was then moved vertically until a position was found where the slot admitted light through the telescope. Having noted the displacement of the card circle with respect to the telescope tube the goniometer was raised or lowered by means of the adjustment screws (5A) until the slot admitted light with the card circle and tube concentric. This was the required setting.

(b) The process was repeated with the slot vertical, the camera being moved to the right or left to obtain concentricity of the card circle and telescope tube.

(c) Finally, the telescope was swung to a position for viewing the end of a diagonal and procedure (b) was carried out but with the camera moved in a direction parallel to its axis, that is forward and back. This result was checked by pointing the telescope at the other end of the diagonal. The axis of rotation of the goniometer now coincided with the centre of the entrance pupil of the lens. Application of this procedure to a camera lens of large minimum aperture, such as that of the Zeiss 12-inch camera, will determine a zone which is elliptical in shape (instead of an exact point) for the pupil position. The mid-position of this zone is accepted.

(7) The aperture was reset to the value used for calibration.

Observing. The calibration observations comprised four rounds of directions, on two different zeros, measured to all targets on each of two diagonals. In some of the observations the centre target was read at the beginning and at the end of each round on a diagonal in addition to the reading taken during the round. This was not done, however, in the example in Table III.

The procedure for reading the targets was as follows:

Diagonal reference	*Observations*
(a) 0909 to 3131	Observed twice by observer A, the first round being from left to right and the second from right to left. Observer B booked the readings. Zero setting approximately 0°
(b) 0909 to 3131	As in (a) but with A and B interchanged and zero setting approximately 90°
(c) 0931 to 3109	As in (a) but with zero setting approximately 180°
(d) 0931 to 3109	As in (b) but with zero setting approximately 270°

For convenience in reduction of the observations the horizontal circle was set to read a few seconds greater than 0° when the centre cross was bisected in the telescope for zero (a). Similarly for zeros (b), (c) and (d) the centre cross was made to read a few seconds greater than 90°, 180° and 270° respectively.

RESULTS

Accuracy. The camera used for the experiment had been calibrated in November 1966 with the Standard equipment. Subsequently, the camera was serviced by Wild. The Wild service included the removal, for cleaning, of the register glass. On reassembly a slight shift might be expected in the position of the register glass and hence the co-ordinates of the point of symmetry might change. On its return by Wild the camera was experimentally calibrated using the T4 and goniometer methods, in December 1966 and January 1967 respectively, and the results compared with the Standard November calibration. Two answers were obtained from both the Standard and T4 calibrations, one computed from two diagonals, corner to corner, and one from these two diagonals plus the two perpendiculars. One answer only, from two diagonals, was obtained using the goniometer method.

Table I shows the values for the distortion corrections at 20-mm. intervals from the point of symmetry, based on the mean of the measured lines of targets (two or four lines as the case may be). Further details of the computations are given in Appendix A.

Observing effort with the Standard Method. Two operators are required for the task, one of whom bisects the targets and books the readings, while the second observes with the theodolite. Two experienced observers can set up and calibrate a camera, observing four rounds on each of two diagonals, in 7 hours.

T4 Method. Two operators are employed on the task, one of whom bisects the targets and books the readings, while the second levels the two levels and reads the vertical arc. The vertical level is very sensitive and much time is spent on levelling it. With experience the time taken to set up and calibrate a camera, observing four rounds on each of two diagonals, is estimated as 12 hours.

Goniometer. Two operators are employed on the task, one observes and the second books the readings. An experienced booker is able to mean the observations during the calibration process. The time taken to set up and calibrate the camera, observing four rounds on each of two diagonals, is 3 hours.

TABLE I

Method and No. of lines	*Final principal distance (mm.)*	*Distortion corrections (μ)*							*Displacement of point of symmetry*	
		Distances from point of symmetry (mm.)							ΔX	ΔY
		20	40	60	80	100	120	140		
Standard 4 lines	152·24	−1	−4	−4	+2	+7	+3	−17	0	+7
Standard 2 diagonals	152·24	−1	−4	−4	+2	+8	+3	−17	0	+7
T4 4 lines	152·24	−1	−4	−3	+2	+7	+3	−18	0	0
T4 2 diagonals	152·24	−1	−4	−3	+2	+7	+3	−18	−2	+3
Goniometer 2 diagonals	152·24	−3	−4	−1	+5	+9	+3	−18	+1	+5
Mean values	152·24	−1·5	−4	−3	+2·5	+7·5	+3	−17·5	0	+4

CONCLUSIONS

Both methods of calibration tested gave adequate accuracy for Ordnance Survey purposes, but the goniometer method was the simpler and quicker. It will be used for all future Ordnance Survey camera calibrations using an improved version of the equipment. There was no significant difference in accuracy between calibrations computed from four lines and those computed from two, and the Ordnance Survey will use only the two diagonals on future calibrations.

APPENDIX A

Mathematical formulae for the calculation of camera calibration by a graphical process

Figure 5 illustrates the goniometer measurements of the centre target and two other targets equidistant from it seen through the camera lens.

R_r and R_l are the correct radial distances of the right- and left-hand targets respectively on the register glass.

R_0 is a nominal value of R approximating closely to R_r and R_l.

A_r and A_l are the angles to the targets observed through the camera lens.

x_r and x_l are the radial distortions.

a_r and a_l are the corresponding angular distortions.

F is the calibrated principal distance (P.D.) of the camera lens.

F_0 is the assigned provisional value of the principal distance.

f is the correction to the principal distance, i.e. $F = F_0+f$.

B_r and B_l are standard angles whose tangents are R_r/F_0 and R_l/F_0 respectively (these angles are not shown in Fig. 5).

O is the mean observed direction.

D_r and D_l are standard directions, and $D_r = B_r$; $D_l = (360° - B_l)$.

T_c, T_r and T_l are *differences*, mean observed direction minus standard direction, at the centre cross and the right and left targets respectively. Note that the standard direction to the centre cross is zero. ($T = (O-D)$; the suffixes indicate the specific target.)

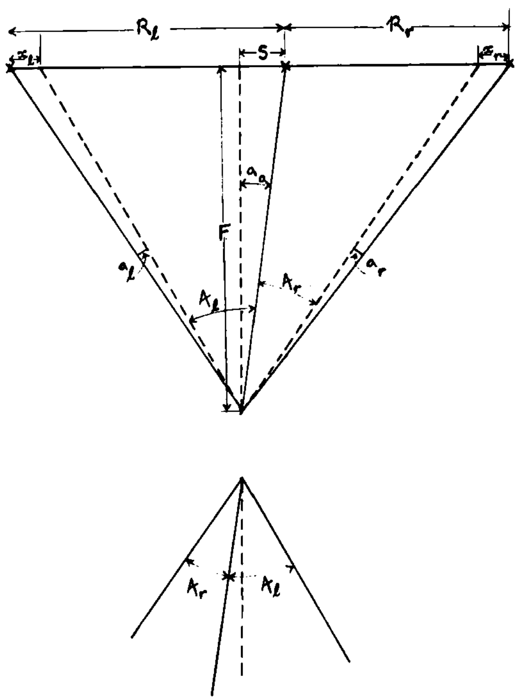


FIG. 5.

The displacement S of the point of symmetry from the centre cross is given by $S = F \tan a_0$.

Referring to Fig. 5:

$$x_r = R_r+S-F\tan(A_r+a_0), \tag{1a}$$

$$x_l = R_l-S-F\tan(A_l-a_0), \tag{1b}$$

$$F = F_0+f. \tag{2}$$

The angle a_0 is small, so approximately

$$\tan(a_i \pm a_0) = \tan a_i \pm (1+\tan^2 a_i)\, a_0. \tag{3}$$

Substituting from (2) and (3) into (1a) and (1b)

$$x_r = R_r+S-F_0\tan A_r-F_0\,a_0-F_0\tan^2 A_r\,a_0-f\tan A_r-fa_0-f\tan^2 A_r\,a_0, \tag{4a}$$

$$x_l = R_l-S-F_0\tan A_l+F_0\,a_0+F_0\tan^2 A_l\,a_0-f\tan A_l+fa_0+f\tan^2 A_l\,a_0. \tag{4b}$$

Again because a_0 is small, approximately

$$S = F_0 a_0 \tag{5}$$

and approximately

$$\tan A_r = \tan A_l = R_0/F_0 = \tan A_0. \tag{6}$$

f is also a small quantity so approximately:

$$fa_0 = 0. \tag{7}$$

Substituting from (5), (6) and (7) into (4a) and (4b):

$$x_r = R_r - F_0 \tan A_r - \frac{R_0 f}{F_0} - \frac{R_0^2 S}{F_0^2}, \tag{8a}$$

$$x_l = R_l - F_0 \tan A_l - \frac{R_0 f}{F_0} + \frac{R_0^2 S}{F_0^2}. \tag{8b}$$

Then

$$\frac{x_r + x_l}{2} = \frac{R_r + R_l}{2} - \frac{F_0(\tan A_r + \tan A_l)}{2} - \frac{R_0 f}{F_0}, \tag{9}$$

$$\frac{x_r - x_l}{2} = \frac{R_r - R_l}{2} - \frac{F_0(\tan A_r - \tan A_l)}{2} - \frac{R_0^2 S}{F_0^2}. \tag{10}$$

Expressing the left-hand side of (9) in terms of a_r and a_l, using the first derivative of $R_0 = F_0 \tan A_0$, namely $x = dR_0 = F_0 \sec^2 A_0\, dA_0$,

$$\frac{F_0 \sec^2 A_0 (a_r + a_l)}{2} = \frac{R_r + R_l}{2} - \frac{F_0(\tan A_r + \tan A_l)}{2} - \frac{R_0 f}{F_0}, \tag{11}$$

then

$$\frac{a_r + a_l}{2\cos^2 A_0} = \frac{\tan B_r + \tan B_l - \tan A_r - \tan A_l}{2} - \frac{R_0 f}{F_0^2} \tag{12}$$

and

$$\frac{a_r + a_l}{2} = \frac{(B_r - A_r) + (B_l - A_l)}{2} - \frac{R_0 f}{F_0^2 + R_0^2}. \tag{13}$$

We have defined

$$D_r = B_r; \quad D_l = 360^\circ - B_l; \quad T = O - D,$$

therefore

$$\frac{a_r + a_l}{2} = \frac{(T_l - T_r)''}{2} - \frac{R_0 f \operatorname{cosec} 1''}{F_0^2 + R_0^2}. \tag{14}$$

For two points equidistant from the point of symmetry $x_r - x_l = 0$, so dividing (10) by F_0, expressing it in terms of a_r and a_l and equating to zero gives

$$0 = \frac{\tan B_r - \tan B_l - \tan A_r + \tan A_l}{2} - \frac{R_0^2 S}{F_0^3}, \tag{15}$$

$$0 = \frac{(B_r - A_r) - (B_l - A_l)}{2} - \frac{R_0^2 S}{F_0(F_0^2 + R_0^2)}, \tag{16}$$

$$0 = \frac{(2T_c - T_r - T_l)''}{2} - \frac{R_0^2 S \operatorname{cosec} 1''}{F_0(F_0^2 + R_0^2)}. \tag{17}$$

Rearranging (17),

$$S = \frac{(2T_c - T_r - T_l)'' F_0(F_0^2 + R_0^2)}{(2R_0^2 \operatorname{cosec} 1'')}. \tag{18}$$

For a small change dA in angle A the corresponding radial displacement dx will be given by

$$dx = F_0 \sec^2 A\, dA \tag{19}$$

or

$$dx = \frac{(F_0^2 + R_0^2)\, dA''}{F_0 \operatorname{cosec} 1''}. \qquad (20)$$

Tables based on formulae (14), (18) and (20) can now be prepared from which to draw up a set of standard curves. For each nominal value for a P.D. there will be one set of curves. Since the Ordnance Survey uses cameras with P.D.s of 6 inches, 6·3 inches, and 12 inches three sets of curves were prepared.

Column 1 of Table II lists the nominal values of the radial distances, R_0, from the centre cross of a line of targets, or réseau crosses. Columns 2, 3 and 4 are prepared using the second term on the right-hand side of formula (14). Columns 5 to 9 show the values of this term, in seconds of arc, for successive changes of 10 microns in f at the given values of R_0. Column 10 is prepared using formula (20) and expresses the relation between linear and angular distortion at successive target positions. Columns 11 to 15 give the angular distortion dA'' for values of dx successively increased by 10 microns. Column 16 is prepared from formula (18) and expresses, in seconds of arc of $(2T_c - T_r - T_l)/2$, the displacement of the point of symmetry for successive values of S at 10-μ intervals.

The Standard Graph. From Table II a set of standard graphs are drawn up, one for each tabulated value of f in columns 5 to 9. This is done on a transparent medium so that the standard graph can be used as an overlay. An example of a standard graph, prepared for a nominal 6-inch focal length camera, is shown in Fig. 6. In this case the graph is designed to determine the distortions of the lens when the correction, f, to the assigned provisional value F_0 of the focal length is $-10\,\mu$. The left-hand side of the curve O–O in Fig. 6 is plotted from the values listed in column 5 of Table II with R_0 (column 1) as abscissae. The right-hand side of the graph is the left-hand side rotated through 180°. The remaining curves, $-50+50$, $-40+40$, ..., $+50-50$, are plotted from the values listed in columns 11 to 15 in Table II. The values are plotted as ordinates with respect to the O–O line as axis. Diagonally opposite quadrants are symmetrical. The values listed in column 16 are plotted as a scale along the central perpendicular S scale in Fig. 6 with its intersection with the O–O curve as origin.

The Distortion (T) *Curve.* Table III is the observation form. (If a separate form is used for each camera, details of the camera/réseau combination, standard directions, F_0, etc., can be printed on the form thus saving booking and computing time.) The values of T in column 8 of Table III are plotted graphically, one curve for each diagonal. (See Fig. 7.)

Fitting the Standard Graph to the T Curve. It is customary to make the distortions zero at a given value of R either side of the centre cross. From the set of standard graphs a graph is selected such that its O–O curve will cut the plotted T curve at the chosen distance R either side of the centre cross. As the standard graphs are constructed for values of f changing by 10 μ, the graph selected in practice is the one which approximates most closely to the T curve at distance R.

The standard graph in the present example is one which gives zero lens distortion at distances of about 120 mm. either side of the centre cross, and is the standard graph for $f = -10\,\mu$. This value of f is entered on the form at Table III as a correction to F_0 to give F. Figure 8 shows the standard graph (a transparency) in registration with the graph of the T values for diagonal 0909–3131 in Fig. 7.

The Point of Symmetry. In Fig. 7 perpendiculars are drawn at 120 mm. either side of the centre cross; these are used to determine the displacement, S, of the point of symmetry in the following way.

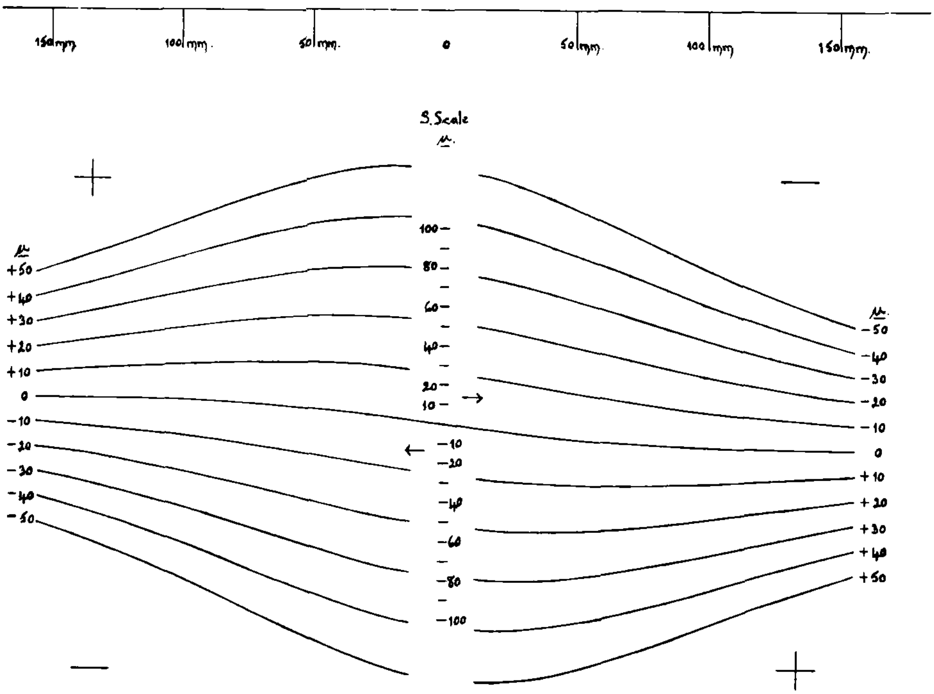
Table II

Standard values for a 6-inch lens. F_0 as 152·15 mm.; $F_0^2 = 23{,}150$. cosec 1″ = 206,265

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
R_0 (*mm.*)	R_0^2	$\frac{\text{cosec } 1''}{F_0^2+R_0^2}$	$\frac{R_0 \text{ cosec } 1''}{F_0^2+R_0^2}$	(4) × 10 μ	(4) × 20 μ	(4) × 30 μ	(4) × 40 μ	(4) × 50 μ	$\frac{F_0 \text{ cosec } 1''}{F_0^2+R_0^2}$	(10) × 10 μ	(10) × 20 μ	(10) × 30 μ	(10) × 40 μ	(10) × 50 μ
14·14	200	8·834	124·9	1·2	2·5	3·7	5·0	6·2	1344	13·4	26·9	40·3	53·8	67·2
28·3	800	8·612	243·6	2·4	4·9	7·3	9·7	12·2	1310	13·1	26·2	39·3	52·4	65·5
42·4	1800	8·267	350·7	3·5	7·0	10·5	14·0	17·5	1258	12·6	25·2	37·7	50·3	62·9
56·6	3200	7·828	442·8	4·4	8·9	13·3	17·7	22·1	1191	11·9	23·8	35·7	47·6	59·6
70·7	5000	7·327	518·1	5·2	10·4	15·5	20·7	25·9	1115	11·1	22·3	33·4	44·6	55·7
84·8	7200	6·796	576·7	5·8	11·5	17·3	23·1	28·8	1034	10·3	20·7	31·0	41·4	51·7
99·0	9800	6·260	619·7	6·2	12·4	18·6	24·8	31·0	952	9·5	19·0	28·6	38·1	47·6
113·1	12,800	5·738	649·1	6·5	13·0	19·5	26·0	32·5	873	8·7	17·5	26·2	34·9	43·6
127·3	16,200	5·242	667·2	6·7	13·3	20·0	26·7	33·4	798	8·0	16·0	23·9	31·9	39·9
141·4	20,000	4·780	676·0	6·8	13·5	20·3	27·0	33·8	727	7·3	14·5	21·8	29·1	36·4
155·6	24,200	4·356	677·7	6·8	13·6	20·3	27·1	33·9	663	6·6	13·3	19·9	26·5	33·1

(16)	
$\frac{R_0^2 \text{ cosec } 1''}{F_0(F_0^2+R_0^2)}$ *for* R_0 *as* 120 *mm.* = 520″	
× 10 μ	5·2
× 20 μ	10·4
× 30 μ	15·6
× 40 μ	20·8
× 50 μ	26·0
× 60 μ	31·2
× 70 μ	36·4
× 80 μ	41·6
× 90 μ	46·8
× 100 μ	52·0

Draw a line in Fig. 7 from the point where the T curve cuts the left-hand perpendicular to the point where the T curve cuts the right-hand perpendicular. The intercept between this line and the T curve, measured on the S perpendicular scale of the standard graph, is S. The direction of S is given by the arrow on the standard graph which points to the reference number of the end target towards which the point of symmetry is displaced. This is done for both diagonals. Finally,



Standard Graph for F 6" lens, and f = −·01 mm.

FIG. 6.

the displacements of the point of symmetry are indicated on the diagram in Table III and resolved into X and Y réseau co-ordinates. Note that the perpendiculars in Fig. 7 must be at the distance of 120 mm. for the purpose of determining S because $R_0 = 120$ mm. was used to calculate column 16 in Table II. The distance to the points of zero distortion in Fig. 7 is, however, at choice when selecting a standard graph. It is a coincidence here that $R_0 = 120$ mm. and that the points of zero distortion are also at about 120 mm.

Distortion Corrections. The values in row d in Fig. 8 are the ordinates of the plotted T curve read relative to the O–O curve on the standard graph. This is done for both diagonals, and the comparable values from the four semi-diagonals are meaned. This gives the final distortion corrections, and these are entered on the form in Table III, column 9. The mean distortion corrections are plotted (see Fig. 7) and values of distortion corrections at 5-mm. intervals of radial distance

TABLE III

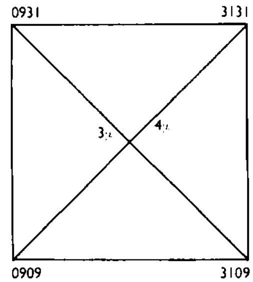
Camera, Type, and No.: Wild, 6 inch, No. 687
Réseau No.: 2653. Filter No. 1897
Aperture: 5·6

Date: 4th January, 1967

Standard P.D. (F_0)	152·25 mm.
Correction f	−0·01 mm.
New P.D. (F)	152·24 mm.

(1) *Réseau cross No.*	*Observed directions* (2) 1st round	(3) 2nd round	(4) 3rd round	(5) 4th round	(6) *Mean observed direction O*	(7) *Standard direction D*	(8) *Mean minus standard T″*	(9) *Corrections* (μ) *(mean curve)*
0909	45 36 34	36 30	135 36 53	36 52	36 42·3	45 37 01·3	−19·0	−45
1010	42 53 07	53 05	132 53 27	53 18	53 14·2	42 53 21·2	− 7·0	−20
1111	39 53 44	53 43	129 54 02	53 56	53 51·3	39 53 45·0	+ 6·3	−2
1212	36 37 05	37 05	126 37 30	37 23	37 15·7	36 36 58·6	+17·1	+7
1313	33 02 07	02 08	123 02 26	02 23	02 16·0	33 01 57·8	+18·2	+9
1414	29 08 04	08 04	119 08 27	08 20	08 13·7	29 07 56·6	+17·1	+7
1515	24 54 49	54 46	114 55 07	55 04	54 56·5	24 54 44·5	+12·0	+2
1616	20 22 57	22 57	110 23 16	23 12	23 05·5	20 23 00·2	+5·3	−2
1717	15 34 15	34 13	105 34 35	34 30	34 23·3	15 34 17·0	+6·3	−4
1818	10 31 26	31 26	100 31 46	31 37	31 33·7	10 31 26·6	+7·1	−3
1919	5 18 22	18 25	95 18 41	18 40	18 32·0	5 18 25·8	+6·2	−2
2020	0 00 00	00 00	90 00 16	00 11	00 06·7	0 00 00	+6·7	0
2121	354 41 36	41 35	84 41 53	41 50	41 43·5	354 41 31·5	+12·0	
2222	349 28 35	28 32	79 28 53	28 48	28 42·0	349 28 33·4	+8·6	*Values*
2323	344 25 43	25 44	74 25 60	25 55	25 50·5	344 25 40·5	+10·0	*at 5 mm.*
2424	339 37 00	37 00	69 37 14	37 10	37 06·0	339 36 59·8	+6·2	*intervals*
2525	335 05 05	05 06	65 05 20	05 14	05 11·4	335 05 14·4	−3·0	0 0
2626	330 51 46	51 46	60 52 02	52 00	51 53·4	330 52 01·4	−8·0	5 −1
2727	326 57 43	57 43	56 57 57	57 55	57 49·5	326 58 03·1	−13·6	10 −2
2828	323 22 43	22 46	53 23 04	23 03	22 54·0	323 23 00·5	−6·5	15 −2
2929	320 06 10	06 11	50 06 29	06 28	06 19·5	320 06 16·5	+3·0	20 −3
3030	317 06 46	06 47	47 07 07	07 08	06 57·0	317 06 39·6	+17·4	25 −3
3131	314 23 18	23 16	44 23 41	23 40	23 28·7	314 22 57·4	+31·3	30 −3
								35 −4
								40 −4
0931	225 36 52	36 58	315 36 40	36 45	36 48·7	225 37 02·0	−13·3	45 −4
1030	222 53 28	53 26	312 53 21	53 14	53 22·3	222 53 19·0	+3·3	50 −3
1129	219 54 05	54 04	309 53 57	53 55	54 00·3	219 53 42·7	+17·6	55 −2
1228	216 37 25	37 26	306 37 19	37 14	37 21·0	216 36 59·5	+21·5	60 −1
1327	213 02 24	02 26	303 02 19	02 16	02 21·3	213 01 59·7	+21·6	65 0
1426	209 08 24	08 24	299 08 15	08 11	08 18·6	209 07 58·6	+20·0	70 +2
1525	204 55 06	55 02	294 54 58	54 54	55 00·0	204 54 46·7	+13·3	75 +3
1624	200 23 16	23 12	290 23 06	23 03	23 09·3	200 22 57·8	+11·5	80 +5
1723	195 34 30	34 29	285 34 22	34 18	34 24·7	195 34 19·5	+5·2	85 +7
1822	190 31 38	31 37	280 31 28	31 28	31 32·7	190 31 28·0	+4·7	90 +8
1921	185 18 38	18 39	275 18 30	18 27	18 33·5	185 18 27·1	+6·4	95 +9
2020	180 00 12	00 12	270 00 03	00 02	00 07·3	180 00 00	+7·3	100 +9
2119	174 41 50	41 50	264 41 41	41 37	41 44·6	174 41 39·6	+5·0	105 +9
2218	169 28 47	28 42	259 28 37	28 32	28 39·5	169 28 34·7	+4·8	110 +8
2317	164 25 57	25 54	254 25 47	25 44	25 50·5	164 25 40·5	+10·0	115 +6
2416	159 37 13	37 09	249 36 59	36 58	37 04·8	159 37 01·0	+ 3·8	120 +3
2515	155 05 16	05 16	245 05 13	05 05	05 12·4	155 05 14·4	−2·0	125 −1
2614	150 51 64	51 57	240 51 50	51 49	51 54·8	150 52 01·4	−6·6	130 −5
2713	146 57 52	57 53	236 57 47	57 46	57 49·5	146 58 01·2	−11·7	135 11
2812	143 22 63	22 58	233 22 51	22 48	22 55·0	143 22 58·7	−3·7	140 −18
2911	140 06 20	06 24	230 06 07	06 05	06 14·0	140 06 14·2	−0·2	145 −25
3010	137 06 59	06 62	227 06 41	06 44	06 51·5	137 06 38·8	+12·7	150 −34
3109	134 23 32	23 35	224 23 22	23 14	23 25·7	134 22 53·4	+32·3	155 −44

Point of symmetry



Displacement of point of symmetry:

ΔX: $+1\mu$

ΔY: $+5\mu$

Observers: W. Sly, A. R. P. Moore

Means: J. Pierpoint

Graph: J. Pierpoint

Checked: M. Tyte

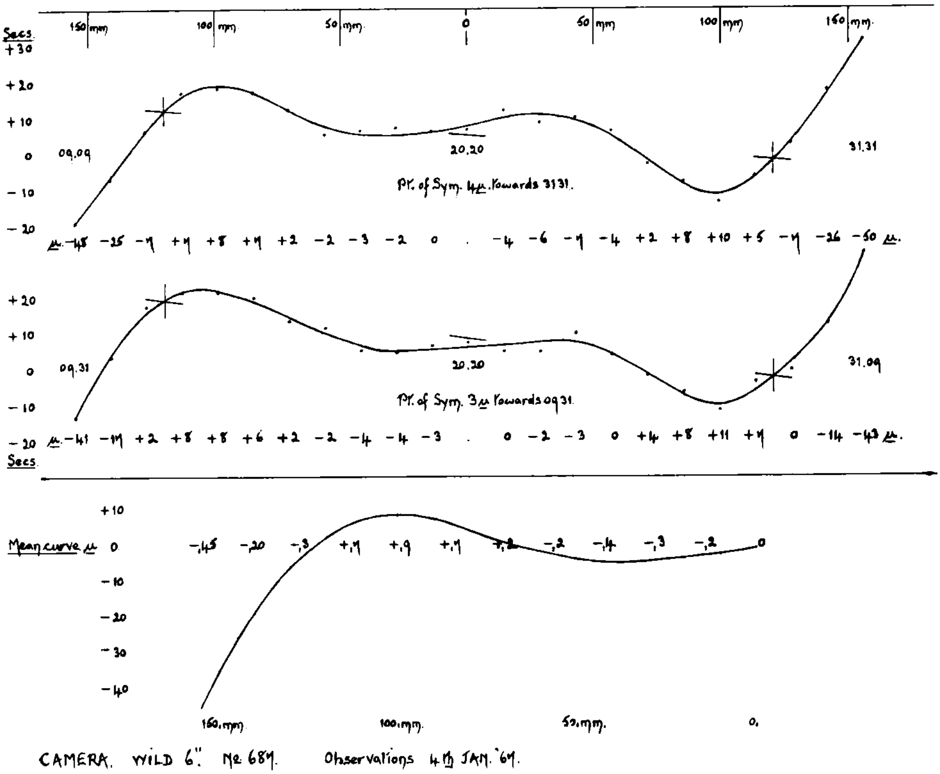

CAMERA. WILD 6". No 687. Observations 4th JAN. '67.

FIG. 7.

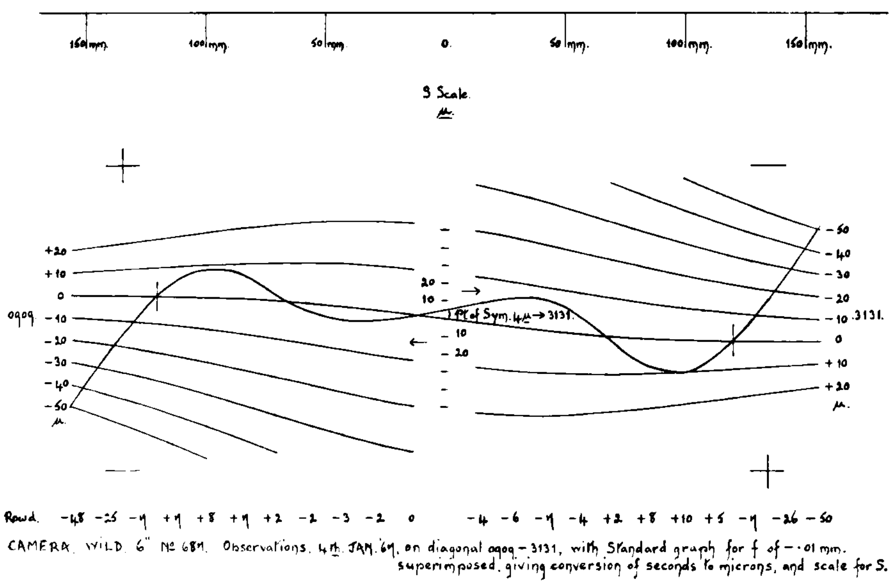

CAMERA. WILD 6" No 687. Observations. 4th JAN. '67. on diagonal 0909 – 3131, with Standard graph for f of – ·01 mm.
superimposed, giving conversion of seconds to microns, and scale for S.

FIG. 8.

from the point of symmetry are interpolated from the mean curve and entered on the form in Table III, column 9.

The Advantage of Graphical Computation. The effort required to produce standard graphs and to compute standard directions is greater than that required to carry out the computations for a calibration by the usual computing process. However, once these aids have been prepared, the time for computing subsequent calibrations is reduced to a few minutes for each camera. Graphical methods may not be justified when electronic computing facilities are available if the calibration can be promptly processed. Otherwise, graphical computation enables a calibration to be completed easily in a day when observations are made on the goniometer.

Acknowledgement. Equations (1a) to (10) in this appendix are based on an analysis supplied by Messrs. Wild Heerbrugg Ltd.

Résumé

La communication décrit deux procédés d'étalonnage des appareils de prise de vues au moyen de mesures faites par des théodolites de haute précision. Les mesures ne déterminent que la distortion radiale, la distorsion transversale étant considérée comme négligeable. L'auteur présente en outre une méthode graphique pour calculer les résultats.