ADJUSTMENT OF HORIZONTAL CONTROL SURVEYS

Paper presented by Australia[1]

1. This paper discusses the adjustment of surveys by the method of variation of co-ordinates, a method which has been found suitable for the adjustment of a wide variety of control surveys. It reviews some of the basic decisions which have to be taken by a survey organization preparing an electronic computer programme for this task.

2. In this paper, control survey means any survey, of any order of accuracy, whose aim is to provide co-ordinated points for the control of mapping. It includes, at one extreme, the geodetic survey of a continent and at the other, a tacheometric traverse or photogrammetric block adjustment. This paper does not discuss the adjustment of heights.

3. In the last decade, the nature of most control surveys has changed. Until 1956, control surveys consisted of a large number of observed angles, which were used to transmit both scale and azimuth between a few scattered base lines and Laplace azimuths, or between points fixed by a survey of higher order. But the measurement of distance is now as easy as the measurement of angles, and modern control surveys are likely to contain a large number of measured lengths. Traversing has tended to replace triangulation and, as the transmission of azimuth by a traverse is relatively weak, the number of Laplace azimuths in a geodetic survey also tends to be much higher than before. Instead of merely adjusting angles, we now have the problem of adjusting angles, azimuths and distances simultaneously.

4. In Australia, for example, the geodetic survey contains 2,506 first-order stations; there are 533 Laplace stations and over 1,720 measured distances. The areas between the chains of geodetic survey are being filled either with second-order tellurometer traverses, containing equal numbers of angles and distances or with aerodist trilateration, containing distances only. This control is in turn broken down by photogrammetric block adjustments, and some tests have been made of least-squares adjustments of angles read on the air photographs in a Wild RT 1 radial triangulator; these adjustments contain angles only. In contrast, in the north-east of Australia and in New Guinea, a HIRAN survey has been made by the United States Air Force, and this adjustment contained azimuths and distances only, with no angles. All these different types of control survey have been computed and adjusted by the same variation of co-ordinates computer programme.

5. Using logarithms or desk calculators, the work of adjusting surveys by the variation of co-ordinate method, especially on the spheroid, was so great that the task was seldom attempted. It is, however, a convenient method for electronic computers, especially the more powerful machines, such as the CDC 3600 or IBM 7090. The programme will not be written in two or three days; it may contain over 1,000 Fortran cards. But the advantages of using an electronic computer for the adjustment of surveys are so great that no survey authority should ever again consider doing an extensive adjustment by hand. If no computer is locally available, the work is better contracted out or performed by some other agency.

Advantages of the method of variation of co-ordinates

6. Apart from its great flexibility, that is to say, the fact that it can be used to adjust any combination of angles, azimuths and distances, the advantages of the method are as follows:

(*a*) No conditions have to be listed; in a complicated survey network, especially one containing measured distances, the formation of the geometric conditions which the adjusted survey must satisfy is a complicated task, which it is difficult to programme for an electronic computer;

(*b*) The adjusted latitude and longitude of each survey station are produced immediately, without further computation, as well as the adjusted angles, azimuths and distances; these latitudes and longitudes can be converted by the computer into eastings and northings, which can be included in the output if desired;

(*c*) There is no limit to the number of observations which can be included in a computer of given size, and new observations are easily added to an old adjustment, and the whole work readjusted, at any time.

[1] The original text of this paper, prepared by A. G. Bomford, Division of National Mapping, Department of National Development, Canberra, appeared as document E/CONF.52/L.30.

7. The need to supply preliminary latitudes and longitudes is sometimes a disadvantage. In triangulation or traverse there is no problem, as an unadjusted traverse can easily be computed to give preliminary co-ordinates for each point. With trilateration, such preliminary computation is less easy, but it suffices to measure co-ordinates as accurately as possible from a map or diagram, and then to iterate the adjustment several times. In photogrammetric surveys, preliminary co-ordinates may be obtained from a slotted template assembly.

On the grid or on the spheroid

8. An adjustment can be performed either in terms of latitudes and longitudes on the spheroid, or in terms of eastings and northings on the grid. When computations were done by hand, the grid had the advantage that the formulae were simpler, and the work much less onerous. On an electronic computer, the saving in both programming and computing, when using these simpler formulae, is trivial. Working on the grid causes difficulty when surveys cross zone boundaries. Except in a small country which lies wholly within one zone of a projection, it is therefore best to adjust on the spheroid. Grid co-ordinates will certainly be required for the adjusted stations, but they are easily included in the output by the direct conversion of the adjusted latitudes and longitudes.

Angles or directions

9. Survey networks can be adjusted by the method of angles or by the method of directions, and the controversy as to which is the better continues undiminished to this day. The method of directions may have some theoretical advantages, but the results obtained by the two methods are similar. The method of angles is the simpler, and some experienced computers claim that in practice it produces better results. In view of this controversy, we may allow the matter to be settled in the direction of simplicity. No survey authority adopting the method of angles is likely to have serious cause to regret it, and angles are therefore recommended.

Outline of the method

10. From the preliminary co-ordinates supplied, the length and azimuth is computed of every line over which an observation has been made. If the observations and the preliminary co-ordinates were perfect, these computed azimuths and distances would accord with the observed angles, azimuths and distances. In practice, they will not accord exactly, and the problem is to find corrections to the preliminary co-ordinates which will minimize the weighted sums of the squares of the differences between the observed and computed values of the angles, azimuths and distances.

11. *Symbols:*

φ = Latitude, positive north
λ = Longitude, positive west
L = The length of a line
A_{12} = The azimuth of a line from station 1 to station 2
$\Delta\varphi$ = A small change in φ, and similarly for λ, L and A
ρ, ν = Radii of curvature of the spheroid in meridian and prime vertical
$P_{12} = (\rho_1 \sin A_{12})/L$
$Q_{12} = (\rho_2 \sin A_{21})/L$
$R_{12} = (\nu_2 \cos A_{21} \cos \varphi_2)/L$
$S_{12} = -\rho_1 \cos A_{12} \sin 1''$
$T_{12} = -\rho_2 \cos A_{21} \sin 1''$
$U_{12} = +\nu_2 \sin A_{21} \cos \varphi_2 \sin 1''$
σ = Standard error
$O - C$ = The observed value of a quantity minus the value computed from the preliminary co-ordinates
A, Z, L = Subscripts pertaining to an angle, azimuth or length

Basic formulae

12. On the spheroid, the basic formulae connecting changes in the azimuth and length of a line with changes in the co-ordinates of the end points are:

$$\Delta A_{12} = P_{12}\Delta\varphi_1 + Q_{12}\Delta\varphi_2 + R_{12}(\Delta\lambda_2 - \Delta\lambda_1) \qquad (1)$$
$$\Delta L_{12} = S_{12}\Delta\varphi_1 + T_{12}\Delta\varphi_2 + U_{12}(\Delta\lambda_2 - \Delta\lambda_1) \qquad (2)$$

In these formulae, ΔA, $\Delta\varphi$ and $\Delta\lambda$ are in seconds of arc.

13. For computing distance and azimuth from the preliminary latitudes and longitudes, the formulae of Robbins (1962) are convenient, and accurate to about 2 cms at 1,600 kms, which suffices for all aerodist and HIRAN lines.

14. For transforming adjusted latitudes and longitudes to transverse Mercator grid co-ordinates, the Redfearn formulae (1948) are correct to better than 1 mm anywhere in a 6° zone. For meridian distance, formula (8.39) in G. Bomford (1962) is correct to 0.5 mm in latitude 45°.

Observation equations

15. In a least-squares adjustment, all the observation equations must have the same dimensions; otherwise the normal equations contain coefficients with mixed dimensions, and the equations are meaningless. In paragraph 12, equation (1) is dimensionless, and equation (2) has the dimension of length.

16. It is well known that the method of least squares provides the most probable corrections to the observed quantities only if the observations are correctly weighted, and in the past this has customarily been achieved by multiplying each observation equation by the square root of the weight of the observed quantity.

17. However, the weight of each quantity is, by definition, inversely proportional to the square of its standard error. It is therefore essential, in all adjustments, but particularly one containing mixed units, to divide each observation equation by the standard error of the observed quantity. In this way we not only make the correct allowance for weights, but make each observation equation dimensionless, and independent of the units used. Provided we express an observed quantity and its standard error in the same units, and divide each observation equation by its standard error, then (and only then) can mixed quantities, of any dimensions, in any units, be rigorously combined in a single adjustment.

18. For each observed angle, by subtracting equation (1) for the line 1–2 from equation (1) for the line 1–3, and dividing through by the standard error of the observed angle, we obtain this observation equation:

$$[(P_{13} - P_{12})\Delta\varphi_1 - (R_{13} - R_{12})\Delta\lambda_1 - Q_{12}\Delta\varphi_2 - R_{12}\Delta\lambda_2 + Q_{13}\Delta\varphi_3 + R_{13}\Delta\lambda_3]/\sigma_A = (O - C)_A/\sigma_A$$

19. For a Laplace azimuth, we use equation (1) for the azimuth line, but include an extra term to satisfy the Laplace equation ($-\Delta\lambda\sin\varphi$) if the longitude of the azimuth station is changed by $\Delta\lambda$:

$$[P_{12}\Delta\varphi_1 - (R_{12} - \sin\varphi_1)\Delta\lambda_1 + Q_{12}\Delta\varphi_2 + R_{12}\Delta\lambda_2]/\sigma_Z = (O - C)_Z/\sigma_Z$$

20. For an observed distance, from equation (2):

$$[S_{12}\Delta\varphi_1 - U_{12}\Delta\lambda_1 + T_{12}\Delta\varphi_2 + U_{21}\Delta\lambda_2]/\sigma_L = (O - C)_L/\sigma_L$$

Along any geodesic $U_{21} = U_{12}$, so that computation of U_{21} can be avoided.

Standard errors

21. In practice, there is little difficulty in assessing adequate values for σ_A, σ_Z and σ_L. For example, on the geodetic surveys in Australia, the following values were used:

For a first-order angle with a Wild T3 on two nights: $\sigma_A = 0''.7$;

For a Laplace azimuth, single ended, with DKM3a or T4, with azimuths and longitudes on two nights: $\sigma_Z = 1''.0$;

For simultaneous reciprocal azimuths: $\sigma_Z = 0''.45$;

For the mean of two independent Tellurometer measurements: $\sigma_L = 0.03$ ms + 3 ppm.

For surveys of lower order, other values can be chosen. It is not even essential that the values chosen be correct; it suffices for them to be in the correct ratio to one another. It is convenient to apply general standard errors in this way, but it is easy to arrange for individual observations which are unusually strong or weak to have their standard errors modified.

Formation of normal equations—band-width

22. It is convenient to arrange for the observation equations to be formed one at a time, and for the squares and products of the coefficients to be computed and added into the matrix of the coefficients of the normal equations immediately. In this way, the number of observations is in no way limited by the size of the computer.

23. The maximum size of an adjustment for a given computer will, however, be limited by the number of unknowns, which is exactly twice the number of points to be adjusted. Thus 100 points require 200 unknowns, and the full matrix of the coefficients in the normal equations would contain 40,000 elements. Since the matrix is symmetric, little over half need be stored, but the situation can be improved much further. In most surveys, observations occur between stations which are relatively close to each other. It is possible that a HIRAN line may extend from one end to the other, but this is exceptional. Usually, if stations are allocated serial numbers which start at one end of the survey and increase steadily to the other end, observations will occur between stations whose serial numbers are not widely dissimilar.

24. When the normal equation matrix is formed, all the non-zero coefficients then tend to lie close to each other in a band parallel to the principal diagonal. The zero coefficients outside this band need not be stored. In a matrix of order N and band-width M (on one side of, and excluding, the principal diagonal) the number of elements in the band matrix is $(M+1)\,(2N-M)/2$. In a typical case, with $N = 150$ and $M = 20$, the number of elements to be stored is 2,940, compared with 11,325 on and above the principal diagonal, and 22,500 in the full matrix.

25. By carefully allocating serial numbers to the variable stations in such a manner that the band-width is as small as possible, large adjustments can be carried out wholly within the high-speed store of a computer, and computation times can be greatly reduced.

Solution of normal equations—direct or iterative methods

26. Direct methods have been criticized on the grounds that, if normal equations are ill-conditioned, round-off errors may accumulate and make the solution worthless. But in survey networks, a strong fix for all stations is one of the surveyor's chief aims, so the equations are never likely to be seriously ill-conditioned. In Australia, Choleski's direct method has been used, and found entirely satisfactory. It is readily modified for use with band matrices, and it is then extremely quick and economical in the use of computer storage. It is nevertheless wise to guard against an occasional unstable matrix by iterating every adjustment at least once.

Fixed points and fixed observations

27. Points are very easily held fixed: $\Delta\varphi$ and $\Delta\lambda$ are simply put equal to zero in any observation equation in which they occur. The method of variation of co-ordinates is thus very suitable for adjusting new surveys in terms of existing control.

28. There is little justification for holding an observed quantity fixed, while the terminal points are free to move; every observation has its standard error, and should receive a corresponding adjustment. However, if it is desired to hold an observation fixed, this can be achieved by giving it an arbitrary standard error artificially close to zero. Division by an exact zero is likely to cause trouble in an electronic computer, as the quotient is infinite.

Listing the data

29. It is convenient to list all observations at one station on one data sheet, on which each line, corresponding to a punched card, contains the observed direction, Laplace azimuth and distance (when the latter exist) to one distant station. Even if the adjustment is to be by the method of angles (see para. 9 above), it is best to list directions in the data, as they can be copied directly from the field book or abstract. The computer can obtain the angles by subtracting each direction from the next in turn.

External angles

30. The question arises whether to form observations with external angles, such as the angles of about 270° which occur at the corners of a braced quadrilateral. On the question whether the inclusion or exclusion of external angles gives the better adjustment, there is still controversy. One advantage of including external angles is that less care has to be taken in the tabulation of the data; the angle to be omitted does not have to be indicated. The advantage of excluding external angles is that there is a valuable reduction in the band-width, and the programme is a little simpler. On the whole, it is considered best to write a programme which does not form an observation equation with the angle lying after the last direction listed. When it is necessary to include the closing angle, as at the centre point of a polygon, then the opening direction can be repeated at the end of the list.

Iteration

31. It is essential to iterate adjustments of this type: first, because the initial set of preliminary co-ordinates may be seriously inaccurate (see para. 7 above), giving erroneous values for P, Q, R, S, T, U; secondly, as a check against instability in the normal equations (see para. 24 above). But there is no need to make iteration wholly automatic. After every adjustment, before iterating, the output should be carefully inspected to ensure that no intolerably large correction has been made, suggesting an error in the data.

It therefore suffices for the computer to punch out cards containing the adjusted latitudes and longitudes in such a form that they can be substituted for the original co-ordinates in the data deck before the adjustment is re-run. As a change in longitude implies a change in Laplace azimuth, cards containing the revised Laplace azimuths need to be punched and substituted in the data deck also.

The Australian "varycord" programme

32. A programme for the variation of co-ordinates on the spheroid has been running in Australia since September 1964. The programme is written in Fortran for the Control Data Corporation 3600 computer owned by the Commonwealth Scientific and Industrial Research Organization in Canberra. The CDC 3600 is a very fast and powerful machine, with a high speed store of 32,000 words, each containing 10 decimal digits. The programme will adjust up to 100 variable points; forty more can be held fixed. From reading the first card of the data to printing the last line of the output, a full-sized adjustment takes about five minutes. This programme was extensively used for the geodetic adjustment of Australia and it has been used to adjust all the different types of control surveys mentioned in paragraph 4 above.

33. The Division of National Mapping would be glad to make listings of this programme available to any other survey organization. A description of the programme has been written and pre-prints can be supplied. Within reason, National Mapping could also assist other survey organizations by running adjustments for them. If the observing authority can tabulate its observations on the special data sheets provided, the actual adjustments can be carried out at little or no cost. Requests for assistance of this sort should be addressed to the Director, Division of National Mapping, Canberra.