Latitude and Longitude Determinations

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Early in 1940 it was decided to map many hundreds of square miles of country in the Newcastle Waters district of the Northern Territory, a country of plains and dense thickets of lancewood, with large patches of desert.

The information desired was the delineation of topographic features and the classification of the different types of country, subdivision being the ultimate object. Precise accuracy was not desired in the work, the margin of allowable error being 200 yards. Compass traversing, combined with motor truck speedometer mileages, were found to give the required accuracy provided a good control point was available at approximately 20 mile intervals.

Astronomical observations obviously had to provide these control points, and it is the purpose of this article to tell the story of these observations and the remarkable results that were obtained by the use of novel adaptations to existing field methods.

Before leaving Darwin for the scene of operations, fairly extensive exercises were undertaken in astronomical observations for both latitude and longitude, the methods used being circum-meridians for latitude, and position lines for longitude, as well as a check on latitude. These experiments gave excellent results, but the time involved made it quite clear that a surveyor working with unskilled help would require an evening and the following day in order to carry out a fixing.

This length of time, and the laborious calculations involved, pointed to the need for quick geographic fixings which could be completed in two or three hours. I think that most surveyors will agree that after traversing all day in the dust and heat of central Australia, two or three hours is quite enough to spend on observations before crawling into the sleeping-bag. With these factors in mind, the following problem was set:

(1) To determine latitude to within 3·0s. of arc.
(2) To determine longitude to within 3·0s. of arc.
(3) To determine true bearings to within 3·0s. of arc.
(4) Time for observations and calculations not to exceed three hours.
(5) Calculations to be simple and self-checking.
(6) Observations to be within the power of a 3½-inch optical micrometer theodolite.

The above six conditions set out the ideal, and it was more in the spirit of recreation than with real hope of achievement that the problem was tackled. After various experiments, a technique was evolved that enabled the following results to be obtained at point L.26 on 11th July, 1940.

1840 hrs.	Observing commenced.
2100 hrs.	Calculations completed, with the following probable errors:

Latitude	01·06s. arc.
Longitude	00·20s. arc.
Bearing	00·36s. arc.

These results were obtained by a surveyor working with a labourer as recorder, and unassisted in the calculations. They appear as the perfect answer to the problem set, and have been repeated on so many occasions that it may be of general interest to trace the theory, experiments and observing procedure that made them possible.

Latitude.

Reviewing the field practice in determining latitude, there are three well-known methods which will give the desired accuracy:

(1) Circum-meridian observations.
(2) Talcott's zenith pairs.
(3) Position lines.

With circum-meridians, we find that the number of pairs required depends on the accuracy desired. Clarke, in his " Plane and Geodetic Surveying ", considers that with a five or six inch micrometer theodolite, the error in latitude derived from a single pair of stars should not be more than 3·0s. of arc, and observations of ten pairs should give the latitude correct to 01·0s. of arc.

This indicates that about five pairs are necessary to obtain the result we desire, and on the basis of 20 minutes to observe each star, the time involved would be four hours after allowing for waiting time. Calculations would require a further two hours, making six hours in all. This is much too long, and the method is not suitable.

Position lines for latitude would necessitate eight observations " extra meridian " to obtain the accuracy required, and as well can be imagined, could not be completed within the time allotted. The calculations are very tedious, and an error can very easily be made.

Talcott's pairs, at first sight, appears to be the ideal method. The observations for a single pair take only a few minutes (or should do so), and the reductions are absurdly simple. This could be done and checked in a very short time. This method was enthusiastically taken up and the selection of the stars commenced. Then the weakness of the method was plainly apparent, the necessary conditions being :

(1) The zenith distance shall not exceed 45°.
(2) The difference in zenith distance of the stars forming a pair must be within the range of the eyepiece-micrometer.
(3) The interval between the times of transit of stars forming a pair should not be less than one minute nor greater than 20 minutes.

It was found that the stars in the Nautical Almanac supplied just one pair in six hours. No doubt another could have been found during the night, but they are scarce enough to render the method impracticable without lengthy research into star catalogues. Regretfully we must discard the method, although not entirely so.

Examining the requirements of Talcott's method we find that conditions (1) and (3) are comparatively easy to satisfy, condition (2) being the most difficult to obtain. The object of condition (2) is to eliminate the effect of refraction, and also to permit the reading of the difference of altitude with a fine-reading eyepiece micrometer.

It is submitted that in the modern optical micrometer theodolite we have an instrument which, in terms of older patterns, is as good as an eyepiece micrometer over the whole range of its circle. It can be relied on to give readings accurate to 01·0s. of arc. The theory was put forward that this type of instrument could be used to observe pairs of stars for latitude, in which the difference of altitude was as great as 10 degrees. By applying refraction coefficients to the observed altitudes, it was reasonable to suppose that the errors due to refraction would be of a small order over a range of 10 degrees. It should be noted that when a star is at an altitude of 50 degrees the alteration in the coefficient for refraction for 1° F. is only 0·13s. of arc.

This reasoning indicated that the observation of stars at meridian transit was a possible solution to the latitude problem. As in the Talcott method, the observation of the transits with the instrument always on the same " face " would eliminate any error in collimation in altitude.

This observation was tried out, and using five pairs of stars the results were as follows :

1st night :	Meridian transits.	Lat.	17° 22′ 25″·73
2nd night :	Circum-meridians	,,	17° 22′ 24″·61
3rd night :	Circum-meridians	,,	17° 22′ 25″·77
4th night :	Meridian transits	,,	17° 22′ 25″·24

These results showed that plain meridian transits give results to within 1·0s. of arc of the circum-meridians, and as they can be observed in one hour and calculated in 15 minutes, the problem as far as latitude was concerned was considered solved.

Longitude.

In discussing the question of determination of longitude, I propose to deal with the determination of local sidereal time. Whatever method of observation is used, the relationship between local time and Greenwich time must be established. Short-wave wireless receiving sets are obtainable very cheaply. and are easily operated. They give access to numerous time signals, and no difficulty should be experienced in keeping Greenwich time with considerable accuracy.

Longitude has always been the bugbear of surveyors making astronomical fixings. In order to obtain an accuracy of 0·20s. of time (03·0s. of arc), it is necessary to take at least eight observations " extra meridian ", and carry out lengthy and tedious calculations. I do not think that I am far in error in stating that it would take twelve hours of continuous work to complete. This expenditure of time was out of the question for the problem in hand.

After some thought, it became obvious that the only point in a star's course that was worth studying was the short time near its meridian transit. During that time it follows a course on which calculations for time can be made by the rules of simple proportion. Reading up text-books on this matter gives the impression that weeks of observations and calculations would be needed to carry out meridian transit observations for time. It seemed hopeless that a surveyor's theodolite could possibly approach the precise results of astronomers. This does not prevent the finding of a variation of the method which would be suitable for field surveyors, and as was stated previously, it is the main object of this article to try and describe one such variation.

The essential requirement of the method for ordinary theodolites is that a straight line be marked out on the ground within 10 minutes of arc of the true meridian. The observation post is situated approximately in the centre of this line, and reference lights are placed at either end. This constitutes the preparation for the observation, and in the case of an observer working with a modern theodolite fitted with self-meaning optical micrometers *can be dispensed with*. Absolute reliance can be placed on the circles on these modern theodolites.

The first experiments were made by taking the times of transit of stars over this line both in the north and south, and applying correction for the error of the assumed meridian. The observations were made on the same " face ", the instrument being rotated on its vertical axis. Adjustment of the horizontal axis and collimation ensured that errors were reduced to a very small amount, and the method of observation made the errors due to those causes opposite in sign for the two stars of a pair. It was hoped that these errors would balance out very closely, but after investigation it was found that although no serious errors are derived from a small collimation error, the dislevelment of the transit axis could easily accumulate an error of 0·20s. secs. of time. Even so, the results showed a remarkable consistency, and the first two of them are worth quoting. The first night on which the method was tried out was at No. 7 Bore, Newcastle Waters.

No. 7 Bore.

1st pair	Longitude	8h.	53m.	30·02s.
2nd pair	,,	8	53	30·41
3rd pair	,,	8	53	30·00
4th pair	,,	8	53	29·47

Newcastle Waters.

1st pair	Longitude	8h.	53m.	37·90s.
2nd pair	,,	8	53	38·16
3rd pair	,,	8	53	37·96
4th pair	,,	8	53	37·96

This method gave such consistent results that it was used to establish four or minor topographical points, and again the same even results were obtained.

Even after obtaining these consistent results the uncomfortable feeling re that the observations were mainly dependent on the careful adjustment of the tran axis and collimation, and were not nicely balanced. The perfect form of observ would be to obtain an observation " face right " and " face left " at the instant the crossed the assumed meridian, and mean the two. This seems an impossible feat, the use of an offset wire in the field of view of the telescope overcomes the di This would consist of a second vertical hair in the telescope, placed so that it sub about 17 minutes of arc in the field of view. In the case of the Wild T.2 theod the vertical stadia wire is admirable.

The vertical stadia wires on the diaphragm of the Wild T.2 are approx 17 minutes of arc on either side of the central vertical wire, giving a stadia con of 1 : 100.

The instrument is pointed to the referring mark, after the vertical axis has *made vertical, with the offset wire on the other side from which the star is approa* The time of transit of the star over the offset wire is taken and the instrument is reve to bring the offset wire over to the other side of the central wire. The time of tra of the star over the offset wire is again taken. The mean of the two transit times gi the true time of transit of the star over the assumed meridian. The central wire only used to keep the instrument pointed on the assumed meridian, and in the case a modern theodolite the circles are used instead. The offset wire being about minutes of arc to one side of the central wire allows sufficient time for the operat of changing face to be carried out.

Several fixings for topographic control were carried out, and I will quote the ca of the observation at L.26 taken on 11th July, 1940. The actual observation consi of observing seven transits, three south and four north (one spare). This was a topo-graphic point, and a fixing was needed after 20 miles of compass and speedometer traverse. Observation commenced at 1840 hours and was completed at 1942 hours. A time signal was received from Greenwich via the B.B.C. at 1730 hours. The calcula-tions were completed before 2100 hours and the results were as follows :

1st pair ..	..	..	Longitude	*8h.*	*54m.*	*59·14s.*
2nd pair ..	..	..	,,	8	54	59·02
3rd pair ..	..	..	,,	8	54	59·04

This method has been tried out very thoroughly and has been checked on long traverses and found very close in its results. The results of some of the later work have been given as an appendix to this article, together with a complete example carried out at Finke Bore, N.T. This complete example is one of the best that I have carried out to date, and it is admitted that the results are not always as good as this. The range of the individual pairs of an observation is generally in the vicinity of 0·3 or 0·4 second of time, and should give a very good determination.

CALCULATIONS.

Having described the steps in the development of the method and quoted some of the results, I had better pass on to the method of reducing the observations.

Consider two stars that transit in the north and south respectively.

Let z_I and z_{II} be zenith distances of N. and S. stars at transit.

δ_I and δ_{II} = declinations of N. and S. stars.

dt_I and dt_{II} = time corrections to N. and S. stars due to error in assumed meridian.

dA = azimuth correction due to error of assumed meridian.

If the assumed meridian coincided with the true meridian, the difference in the stars' right ascensions would be exactly equal to the time interval registered by the chronometer in the observation. This is very rarely the case, and corrections to the times of transit for each star must be calculated and applied. When these corrections have been applied, we have the star's right ascension (local sidereal time) in relation to the chronometer time, the error of which is known in respect to Greenwich time, the relation of which is known in respect to Greenwich sidereal time. The difference between local sidereal time and Greenwich sidereal time is of course the longitude.

When a star is very close to the meridian, the following relationship holds good.

If dA be small,

$$dt_I = dA \sin z_I \sec \delta_I \quad \ldots\ldots \quad (1)$$

$$dt_{II} = dA \sin z_{II} \sec \delta_{II} \quad \ldots\ldots \quad (2)$$

Dividing (1) by (2)

$$\frac{dt_I}{dt_{II}} = \frac{\sin z_I \sec \delta_I}{\sin z_{II} \sec \delta_{II}}$$

whence $$dt_I = \frac{T.\sin z_I \sec \delta_I}{\sin z_I \sec \delta_I + \sin z_{II} \sec \delta_{II}}$$

$$dt_{II} = \frac{T.\sin z_{II} \sec \delta_{II}}{\sin z_I \sec \delta_I + \sin z_{II} \sec \delta_{II}}$$

where $T = dt_I + dt_{II}$ = amount of time obtained by subtracting the difference of R.A.'s of N. and S. stars of a pair from the difference of chronometer times of transits of N. and S. stars of the pair.

By adding (1) and (2) and transposing

$$dA = \frac{15.T}{\sin z_I \sec \delta_I + \sin z_{II} \sec \delta_{II}}$$

seconds of arc, if T is expressed in seconds of time.

In the above formula it is seen that when T is known, the values of dt_I, dt_{II} and dA are obtained by the rules of simple proportion. A slide rule will give the necessary accuracy.

The sign of the correction to be applied to the transit of each star is obtained by the following rule:

If the chronometer time is too small, the correction to the leading star is negative, and conversely, if the chronometer interval is too large, the correction to the leading star is positive. The sign of the correction to the following star is always opposite to that of the leading star.

The complete calculations are as in the following example.

Problem.

At a place in south latitude the times of transit of two stars, β Centauri (in south) and α Bootis (in north), were observed over an assumed meridian. Using a sidereal chronometer, the results were as below.

	α Bootis.	β Centauri.
Chronometer time of transit	19h. 21m. 15·54s.	19h. 08m. 02·32s.
z at transit	37° 02′ 00″	42° 31′ 00″
Declination	N. 19° 29′ 38″	60° 05′ 04″
Right ascension	14h. 12m. 57·31s.	13h. 59m. 38·39s.

Temperature : 50° F.

Chronometer was losing 0·64s. per hour and at 19 hours chronometer time was 14h. 03m. 19·12s. fast on Greenwich sidereal time.

Determine latitude, longitude and bearing of assumed meridian.

Longitude.

β Centauri	19h.	08m.	02·32s.	chronometer time of transit.
α Bootis	19	21	15·34	,, ,,
		13	13·02	difference in chronometer times
			00·14	chronometer loses in 13 minutes
		13	13·16	observed interval
β Centauri	13h.	59m.	38·39s.	right ascension
α Bootis	14	12	57·31	,, ,,
		13	18·92	difference
		13	13·16	observed interval
			05·76s.	chronometer time too small

β Centauri		α Bootis
0·6758	sine z	0·6023
2·0050	secant δ	1·0610
1·357	product of above	0·638

Sum of products = 1·995.

$$\frac{05 \cdot 76}{1 \cdot 995} \times 1 \cdot 357 = 3 \cdot 92\text{s.} \qquad \frac{05 \cdot 76}{1 \cdot 995} \times 0 \cdot 638 = 1 \cdot 84\text{s.}$$

$$dA = \frac{15 \times 5 \cdot 76}{1 \cdot 995} = 43 \cdot 3 \text{ secs. of arc.}$$

β Centauri.				α Bootis.		
19h.	08m.	02·32s.	chronometer time of transit	19h.	21m.	15·34s.
		03·92	corrections			1·84
19	07	58·40	chronometer time true transit	19	21	17·18
14	03	19·03	chronometer ahead of Greenwich	14	03	18·89
05	04	39·37	Greenwich sidereal time	05	17	58·29
13	59	38·39	right ascension	14	12	57·31
08h.	54m.	59·02s.	Longitude	08h.	54m.	59·02s.

Latitude.

β Centauri		α Bootis
47° 29′ 00″·00	altitude	52° 58′ 00″·00
52″·9	refraction	43″·5
47° 28′ 07″·1	corrected altitude	52° 57′ 16″·5
60° 05′ 04″·0	declination	19° 29′ 38″·00
107° 33′ 11″·1	sum	72° 26′ 54″·5
90° 00′ 00″		90° 00′ 00″·0
17° 33′ 11″·1	Latitude	17° 33′ 05″·5

Mean latitude: 17° 33′ 08″·3.

Azimuth.

In the longitude computation it was found that dA was 43·30s. and a correction was added to the northern star to get correct time of transit. This indicates that the bearing of the assumed meridian was 00° 00′ 43″·30s.

In all the practical work which has been done up to date, a mean time chronometer was used, and the calculations for longitude as a result are slightly different. A similar example to the previous one would be worked out as follows, if using a mean time chronometer :

β Centauri	19h.	08m.	02·32s.	chronometer time of transit
α Bootis	19	21	13·17	,, ,, ,,
		13m.	10·85s.	difference (mean time)
			02·17	convert to sidereal time
			00·14	chronometer's losing rate
		13m.	13·16s.	difference of transits (sidereal)
β Centauri	13h.	59m.	38·39s.	right ascension
α Bootis	14	12	57·31	,, ,,
		13	18·92	difference
		13	13·16	chronometer interval
			05·76	chronometer interval too small

β Centauri.		α Bootis.
0·6758	sine z	0·6023
2·0050	secant δ	1·0610
1·357	product of above	0·638

sum of products = 1·995

$$\frac{5\cdot76}{1\cdot995}\times 1\cdot357 = 3\cdot92\text{s.} \qquad \frac{5\cdot76}{1\cdot995}\times 1\cdot357 = 1\cdot84\text{s.}$$

$$dA = \frac{15\times 5\cdot76}{1\cdot995} = 43\cdot3 \text{ secs. of arc.}$$

19h.	08m.	02·32s.	chronometer time of transit	19h.	21m.	13·17s.
		03·92	corrections			01·84
19	07	58·40	chronometer time of true transit	19	21	15·01
09	18	55·98	chronometer ahead of Greenwich mean time	09	19	55·84
09	48	02·42	corresponding Greenwich mean time	10	01	19·17
	01	36·61	convert to sidereal time		01	38·77
19	15	00·35	sidereal time, Greenwich mean time of date	19	15	00·35
05	04	39·38	corresponding Greenwich sidereal time	05	17	58·29
13	59	38·39	right ascension	14	12	57·31
08h.	54m.	59·01s.	Longitude.	08h.	54m.	59·02s.

Latitude and bearing of assumed meridian are calculated as in the previous example.

It is quite in order to accumulate a difference of 0·01 or 0·02s. when reducing a pair of observations, but a greater difference will indicate an error in calculation.

Conclusion.

There is ample time when observing for the meridian altitude to be observed in between the two transits of the star over the offset wire, and it has been my practice to use the same stars for both latitude and longitude.

The method has proved so consistent that it has been used to obtain fixings requiring a considerable degree of accuracy. The example shown in the appendix is one of these and has been completed with the use of stars listed in the abridged list of the Nautical Almanac.

APPENDIX.

Comments on Observing Procedure.

Chronometers.

At first glance it would seem that sidereal chronometers would be much more suitable than mean chronometers. For a surveyor working without skilled assistance, the slight shortening of the calculations which results would be outweighed by the possibility of a constant error in the calculations. The comfortable fact that the chronometer registers local standard time makes it much easier to keep track of time signals. In addition, stop watches are generally rated to mean time.

Chronometers carry very well in motor trucks when placed on the driver's seat with the gymbals locked. The variation in daily rate, due to shocks when travelling over rough country, has never exceeded one second.

Time Signals.

Time signals are given very frequently from Greenwich, over the Empire short wave stations, and it is now possible to obtain a signal every hour from 1630 hours to 2330 hours. They consist of six dots superimposed on the broadcast, the commencement of the final dot being the exact hour. They should never be more than a few hundredths of a second in error. Rhythmic signals would be far superior, but are practically unobtainable with the normal short-wave receiver. The question of time lag from Greenwich is a subject on which I have been unable to obtain much data, but from months of observation it appears that there is little appreciable difference between the Melbourne signals and those from Greenwich. Assuming a time lag for Darwin at 0·06 secs., it would not vary more than 0·02 secs. from that figure for the whole of Australia. It can be safely ignored, as the error introduced will be relative for all points.

Stop Watches.

A very excellent type of stop watch has been used for this work. It is graduated into 10 seconds around the dial, and hundredths of a second can be easily estimated. In using the stop watch to pick up the time signals, the watch is started on the sixth beat. The hands should keep rhythm with the preliminary beats, and by the time that the sixth dot arrives coincidence is easily obtained. After starting the watch, the comparison is made by stopping the watch at an even five or ten seconds mark on the chronometer. It is important that this last operation should be done by ear and not by eye, the rhythm of the chronometer being picked up in a similar manner to the time signal. Merely glance at the chronometer to pick up the count, and continue to count mentally whilst listening to the ticks.

Theodolite.

The best results have been obtained with the instrument on the tripod. The average shift in azimuth over several hours is about 10″ of arc. Pairing of the transits as closely as possible in time reduces the shift over the period of observation to a very small amount.

Pegs should be driven into the ground and bored with holes to receive the tripod. In setting up the instrument, care should be taken that two of the foot-screws are parallel to the meridian. This allows the third screw to make adjustments to the vertical axis. No attempt should be made, prior to the observation, to make the cross bubble traverse, as any disturbance to the bubble makes it unreliable for a short time. As long as the bubble remains in the same portion of the tube on both faces of the instrument the vertical axis is vertical.

Programme.

It is unfortunate that the list of stars has been reduced in the Nautical Almanac, as the paucity of stars extends the time required for the observation. If, however, a copy of the publication " Apparent Places of Fundamental Stars " for the current year is available, the observation time is quite short.

The schedule for observation can be prepared for the whole twenty-four hours of the day, which covers a year's observation. The only alteration for a new station is a change of zenith distance. As the seasons advance, new portions of the twenty-four hour schedule are used. To facilitate rapid changing of face during transits, the recorder should be supplied with the approximate circle readings. As many small stars may be used, setting is quicker than searching.

Setting out the Meridian.

This is easiest carried out by approximation. The longitude is estimated and the chronometer time of the transit of a star is calculated. At this exact instant the instrument is set on the star and clamped at zero. Observation of a transit in the opposite direction will enable a fairly accurate calculation of the error in the assumed meridian to be made. Very rarely will the error exceed 10″ of arc after correcting by this method, which can be carried out very early in the evening when the atmosphere is not sufficiently steady for finer observation. When using a modern theodolite, absolute reliance can be placed on the circles, as it has been found that the unsteadiness of a referring light close to the ground is greater than the uncertainty of a setting made with the optical micrometer. The referring light is excellent as a guard against any major movement, and must necessarily be used when azimuth is required.

Observation.

It is recommended that vertical angles should be read with the instrument face left. The readings taken in this position are true zenith distances in most instruments, and do not need conversion. Thus the early stadia should be read face right, the instrument changed to face left and the zenith distance read at transit. The instrument remains at face left for the late stadia. Between pairs of transits the vertical axis should be checked.

The usual precautions observed when taking trigonometrical angles should be adhered to, and do not need repetition. The calling back of the angles by the recorder is most important.

Equatorial Value of Stadia Wires.

The time taken for a star to travel from stadia wire to stadia wire can be calculated from the rule " angle subtended by wires multiplied by secant declination ". This gives a useful check on doubtful observations, and is an indication of the time available for changing face.

Calculations.

As the position obtained would probably have to be used on the following day. I think it advisable to work out each pair individually, making no attempt to use an epoch for the whole set. Similarly the correction from chronometer time to Greenwich mean time should be scaled from a graph. Using this procedure, each pair checks itself, and the pairs check one another. Final trimming of the calculations can be carried out at a later date at the office.