NEW DETERMINATIONS OF THE VELOCITY OF RADIO WAVES

Carl I. Aslakson

Abstract--The author deduces a new value \(299,794.2 \pm 1.4 \text{ km/sec}\) for the velocity of propagation of radio waves in a vacuum, as compared to the value of \(299,792.4 \pm 2.4 \text{ km/sec}\) which he had previously reported. Whereas both values were determined by comparison of shoran and geodetic measurements of distances between widely separated points, the new value results from comparisons with improved shoran equipment. The improvements are discussed, with emphasis on the introduction of a gain-riding technique to reduce signal intensities to a constant level and eliminate the possible uncertainties in the signal-intensity corrections which were applied in the earlier determination. The methods of making shoran measurements are outlined and the results of a least-square adjustment of the new shoran data are discussed. In view of the corroborative evidence from other recent determinations, the author recommends abandonment of the Birge statistical value of the velocity of light and states that a value of 299,793 \text{ km/sec} has been adopted for current shoran measurements.

In 1949 the writer [ASLAKSON, 1949] reported evidence that the BIRGE [1942] statistical value of the velocity of light was too low. That evidence was obtained from a comparison of conventional triangulation with shoran distances. The value derived was 299,792.4 \pm 2.4 \text{ km/sec}. One unsatisfactory feature of this earlier work was the necessity of applying an empirical signal-intensity correction. Although the writer had considerable confidence in the validity of the correction, it was noted that when the signal intensity was ignored, a velocity very close to the Birge value resulted. Since that time great efforts have been made to remove that source of error and those efforts have met with considerable success.

In February and March 1950, the United States Air Force completed an extensive project in Florida, wherein modified shoran equipment was tested. The shoran modifications and improvements were:

1. Improvement of the goniometer or phase shifters
2. Improvement of the zeroing of circuits
3. Addition of a 0.2-mi sweep to permit more accurate pip alignment
4. Photographic recording of the 0.2-mi oscilloscope, enabling pip alignment errors to be corrected
5. Adoption of a "gain-riding" equipment to permit manual control of the gains, both in the airborne equipment and at the ground station

The last-named improvement was by far the most important one and it effectively removed the troublesome signal-intensity correction. On the bench, this gain control reduced the shoran readings to the same level over ranges far greater than are ever encountered on shoran missions. In the air an extra operator controlled the gains at all times during the line-crossing procedure. The ground-station operators likewise maintained the gain at a constant level. During rapid fluctuations of the signals considerable operator skill is required to "ride the gains," but nevertheless the effectiveness of the technique was reflected not only in the more uniform "sum-distance" curves but in the final computed results. These improvements will be discussed farther on in this report in greater detail.

Shoran method of distance measurement--For the benefit of those who are unfamiliar with the shoran methods as developed in the United States Air Force under the direction of the writer, the following description of the "line-flying" procedure is presented. The shoran instrument is well known and its technical description will not be included here. Basically, shoran is a system in which the travel time of a pulse from an airborne transmitter to a ground transponder and back is measured in the aircraft by means of a goniometer or phase-advancing system. The frequency is controlled by temperature-controlled crystals at the ground stations. The frequency of the crystal now being used is \(93,109.87 \text{ cycles/sec}\), and is chosen because its 537th, 1074th, and 1611th harmonics are \(5\times10^7\), \(10\times10^7\), and \(15\times10^7\) respectively, facilitating comparison with the standard frequencies of WWV. The principle of the shoran design is satisfied for distance when the timing frequency in cycles/sec is equal to one-half the velocity in mi/sec or
where \( f_t \) is the frequency of the timing crystal and \( V_t \) is the velocity of the impulse in mi/sec.

The velocity of propagation in air at a given instant is given by

\[
V = \frac{cK_1}{f_J} \quad \text{(2)}
\]

where \( V \) is the velocity of propagation in air, \( c \) is the velocity in a vacuum, \( K_1 \) is the dielectric constant of the air, and \( \mu \) is the index of refraction. The values of \( K_1 \) and \( \mu \) are dependent on experimental data and will be discussed later in this paper. The Birge statistical value of \( c = 299,776 \) km/sec corresponds to \( 186,271.8 \) mi/sec in units in the United States. The correct crystal frequency corresponding to the Birge velocity would have been \( 186,271.8/2 \) or \( 93,135.9 \). Therefore, it is necessary to apply a correction for the difference between these two frequencies or

\[
\Delta T = \frac{(93,135.9 - 93,109.87)}{93,109.87}
\]

where \( \Delta T \) is the ratio by which the distances in miles were increased in the goniometer design to give the correct distance according to the Birge velocity. The timing-crystal frequency is not to be confused with the transmission frequencies. In shoran the airborne transmitter is usually on a frequency of about 220 to 240 mc while the ground stations reply on a common frequency of about 300 mc.

**Method of distance measurement**—In the geodetic application the distances between shoran ground stations were determined in the following manner: The minimum distance between two stations was obtained by flying across the line between the stations. The counter readings, giving the distances to each ground station were photographically recorded at two-second intervals. The two ground stations involved in a particular line crossing are designated as "rate" and "drift" in accordance with the usual shoran terminology. The minimum sum distance was then determined analytically from a large number of simultaneous observations on each of the two distances, 30 to 40 sum distances being used in each computation. At least 12 and frequently 16 to 18 such measurements were made on each line. Each shoran distance was then corrected by a timing correction and a velocity correction.

The two corrected minimum shoran distances were next reduced to the geodetic distances by geometric corrections involving the altitude of the shoran aircraft and the altitude of the ground stations. The final result was the geodetic distance between stations which was then compared with the geodetic distance as determined by conventional triangulation.

The details of the correction system have been discussed in numerous papers [ASLAKSON and RICE, 1946; ASLAKSON, 1949; ASLAKSON and FICKEISSEN, 1950; ASLAKSON, 1951]. They have been proven in practice and will not be repeated here. However, inasmuch as certain physical measurements are involved in the velocity correction, it is desirable to mention that correction briefly.

**The velocity correction**—As the shoran ray travels between airplane and ground station it passes through an atmosphere of varying barometric pressure, temperature, and water-vapor pressure. The index of refraction changes continuously along the path, with consequent variations in the velocity of the ray. For this reason it is necessary to correct the measured distance for these changes in velocity from the standard velocity incorporated in the design by the timing-crystal frequency.

The changing index of refraction has a twofold effect on the measured distance: (a) a length change due to the different degrees of bending, and (b) a change in velocity along the path which is a function of the index of refraction in accordance with Eq. (2). In shoran work the lapse rates of barometric pressure, temperature, and water-vapor pressure are observed along the approximate ray path by a second airplane and the data are available for determining the correction. KROLL [1949] has shown that it is possible to combine both effects in a single velocity correction by integrating the basic differential equation of terrestrial refraction along the ray path. However, in shoran work a more practical and far less laborious method has been used and comparisons with the Kroll method show that the differences are generally insignificant. In shoran work the effect of the ray bending is separated from the velocity change by assuming a curved path and combining the bending effect with the geometrical correction. The effect due to change of velocity along the path can then easily be shown to be represented by the relationship
where \( \Delta S \) is the velocity correction, \( \Delta \mu \) is the index of refraction minus unity, computed at appropriate intervals along the ray path, and \( ds \) is the corresponding distance increment computed at appropriate intervals along the ray path. In practice the above expression is evaluated by numerical integration. It is combined with the frequency correction in a single expression of the form

\[
\text{Velocity correction in miles} = \left\{ \frac{(V_0 - f)/f}{f} \right\} \times 10^6 \quad \ldots \ldots \ldots \ldots \ldots . (3)
\]

where \( V_0 \) = the value of \( c \) or the in-vacuo value of velocity adopted, \( f \) = the shoran timing-crystal frequency, \( k \) = the altitude of the ground station, \( H \) = the altitude of the shoran aircraft, \( \Delta \mu \) = the increment of the index of refraction between the points selected for the numerical integration, and \( S_a \) = the computed minimum distance from the airplane to the ground station after correcting for the zero readings. Tables giving the distance increments on the various ray paths have been prepared for a large number of paths. Their use and the arrangement of calculations in a compact computing form make the entire integration process simple and straightforward.

The value of \( \mu = 1.0002835 \) for standard dry air at 760 mm pressure and 0°C temperature was originally used in the computation. An extensive investigation of all work which had been
Table 1--Comparison of geodetic distances with shoran determinations based on a velocity of 299,776 km/sec

<table>
<thead>
<tr>
<th>Line no.</th>
<th>No. of crossings</th>
<th>Ground sta. no.</th>
<th>Distances</th>
<th>Shoran - geodetic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Rate</td>
<td>Drift</td>
<td>Geodetic mi</td>
</tr>
<tr>
<td>1-2</td>
<td>18</td>
<td>3</td>
<td>2</td>
<td>45.8884</td>
</tr>
<tr>
<td>2-3</td>
<td>14</td>
<td>2</td>
<td>3</td>
<td>133.0003</td>
</tr>
<tr>
<td>3-4</td>
<td>12</td>
<td>1</td>
<td>2</td>
<td>100.3098</td>
</tr>
<tr>
<td>5-3</td>
<td>12</td>
<td>1</td>
<td>2</td>
<td>134.5003</td>
</tr>
<tr>
<td>5-4</td>
<td>11</td>
<td>1</td>
<td>2</td>
<td>139.1003</td>
</tr>
</tbody>
</table>

The interpolation equation currently in use to correct to the accepted velocity is

\[
\Delta \mu \times 10^6 = 287.6 - 77.54 \frac{P}{T} + 9.66 \frac{e}{T} - 37.84 \frac{e}{(T/100)^2}
\]

where \(\Delta \mu\) = the change from the standard dry-air condition to the observed condition, \(P\) = the total pressure in mb, \(e\) = the water-vapor pressure in mb, and \(T\) = the temperature in °K.

The equation used in computing the values of \(\Delta \mu\) in this paper differed very slightly from the above, the principal change being the change of the index of refraction for standard conditions, and corrections to distances were made for this change.

The distances measured--The project as a whole is shown in Figure 1. All stations were at or near triangulation stations of the U. S. Coast and Geodetic Survey. The inversed geodetic distances were computed using a modified Helmert method and are numerically exact to 0.0001 mile. The results of these measurements are tabulated in Table 1. Examination of the last column clearly shows the increase of the discrepancy with distance, which the writer attributes to the basic velocity error. However, the discrepancy in this column also includes a small constant error which was known to be about -0.0050 mile. In other words, each shoran ground station was known to measure about 0.0025 mile too short. The determination of these errors will be discussed later.
The internal consistency of the individual shoran line crossings was very high as shown by Table 2, which also illustrates that the repetitional accuracy of missions flown on different days is approximately the same as the internal consistency of a single mission. Numerous test missions which were not included in the record missions were flown. One of the most interesting ones was Mission 9-F, illustrated in Figure 2, which brought out three facts: (a) the validity of the gain-riding technique for removing the signal-intensity error, (b) the validity of the timing, velocity, and geometrical corrections, and (c) the first evidence of a multipath propagation effect.

Regarding (a) it is noted in Figure 2 that Line 3-4 was flown at 18 elevations at approximately 1000-ft intervals. In flying in this manner wide variations of signal intensity were encountered as shown in the figure. Yet there is no evidence that the absolute measurement was changed by this signal-intensity range.

Regarding (b) the successive corrections are applied to the original sum-distances with the final reduced distance resulting for each altitude. Again no correlation of reduced distance with altitude is encountered.

However, regarding (c) it is clearly evident that there is a correlation of aircraft heading with measured distance. The headings are shown for each reduced distance. It was clearly evident that this was not related to the signal intensity, and after much study it was demonstrated that this was a multipath propagation effect due to a combination of circumstances seldom encountered in practice. On this mission, the altitude of the airplane was such that a third re-radiated ray from a portion of the B-29 wing was responsible for this effect. The combination of circumstances resulting in such a condition is that, first, the ray re-radiated from the wing must be stronger in signal intensity throughout the crossing than the signal resulting from the phase combination of the direct ray and the ray reflected from the surface of the Earth. Secondly, this ray, which has a slightly longer path length, cannot exceed that path length by more than approximately 0.002 mile or its signal will be obliterated by the signal from the direct ray. Clearly, a mean of all crossings will differ little from a true value. However, a study of the antennae wiring and their placement was made, and before the record missions certain modifications were made in the equipment such that there is no evidence of this effect in the record missions. Indeed, a
portion of the directional effect may be attributable to the fact that these crossings were all near perpendicular crossings rather than the accepted figure-8 technique. In perpendicular crossings, a slight slackness in the gear trains of the goniometer might cause an apparent directional effect. This is eliminated in the figure-8 technique. An observer’s personal error may also contribute to the effect.

Multipath propagation effects from surfaces other than those on the aircraft and the surface of the Earth can cause no concern when shoran methods are used. The path length is so much greater that it is obscured by the direct ray or appears only momentarily on the oscilloscope as a stray, whereas the resultant of the direct and reflected rays registers continuously on the oscilloscope.

Computation of the velocity--The results in Table 1 were analyzed to determine the velocity compatible with the distances obtained. It is also possible in this computation to make an analytical determination of the constant errors of each ground station. It is noted that only three shoran ground stations were used in the work. Inasmuch as 15 distances were measured, there are 15 observation equations in all. These equations are of the form

\[ FS + K_1 + K_2 + K_3 + E = 0 \]

where \( F \) is the factor by which the shoran distances \( S \) (or the design velocity of 299,776 km/sec) must be increased to obtain the correct distance; \( K_1, K_2, \) and \( K_3 \) are the respective constant errors of ground stations 1, 2, and 3; and \( E \) represents the value of \( S-G \), or the shoran reduced distance minus the inversed geodetic distance.

A least-square solution of the above 15 observations results in the following:

\[
F = 1.00000606 \pm 0.0000046 \\
c = 299,776 F = 299,794.2 \pm 1.4 \text{ km/sec} \\
K_1 = -0.00193 \pm 0.00058 \text{ mile} \\
K_2 = -0.00369 \pm 0.00073 \text{ mile} \\
K_3 = -0.00200 \pm 0.00053 \text{ mile}
\]

Thus

\[
K_1 + K_2 = -0.0056 \text{ mile} \\
K_1 + K_3 = -0.0039 \text{ mile} \\
K_2 + K_3 = -0.0057 \text{ mile} \\
\text{Probable error of a shoran measurement} = \pm 0.00118 \text{ mile}
\]

The values of the combinations of the constant ground-station errors must be applied to the shoran measured distances. The values obtained in the solution are negative. The errors are of opposite sign and the shoran distances must be increased by the amount of the factor \( F \) and the constant errors to give the correct distances. The velocity designed into the instrument must also be increased by the factor \( F \).

Determination of local survey error by shoran measurements--An interesting example of the confidence of the writer in the shoran measurements occurred during the course of the project. Examination of Figure 1 shows that five distances are measured to each point. When the shoran measurements were first compared with the geodetic distances, the results at Station (4) near Key West were not compatible with the results of the remaining 10 lines. This discrepancy is best illustrated by reference to Figure 3. Therefore, a preliminary adjustment was made assuming a single constant error for all combination ground stations and the amount of movement of Station (4) necessary to bring it into agreement with the remaining shoran distances was computed. The following observation equations were used.

For lines not including Station (4) the equations took the form

\[ FS + K + E = 0 \]

For lines including (4), two additional terms were added to allow for shift of Station (4) and the equations were of the form

\[ FS + K + d \cos (\alpha - \beta) + E = 0 \]
Fig. 3—Shoran distances minus geodetic distances prior to resurvey of Station 4 at Key West

F, S, K, and E have been defined, d is the distance station (4) should be moved, \( \alpha \) is the azimuth of the line from Station (4) to the remote ground station, and \( \beta \) is the azimuth of the short distance joining the new position of (4) to the original position.

This adjustment resulted in the following determinations: \( d = 0.0067 \text{ mile} \) and \( \beta = 219 \, \frac{3}{4}^\circ \), or \( \beta - 180^\circ = 39 \, \frac{3}{4}^\circ \).

As a result of the above computation, the writer in a written report recommended that a resurvey be made at Key West. This resurvey was made several months later by the U. S. Coast and Geodetic Survey. The error predicted by the writer in the above adjustment was verified and the results were: \( d = 0.0069 \text{ mi} \) and \( \beta = 37.0^\circ \).

Thus a local survey error was predicted within two feet by five shoran measurements which varied from 96 to 320 miles. This instance supports the value of the velocity of radio waves derived from shoran measurements.

In fairness to the local survey party it should be stated here that the error was not in their field work but was caused by the fact that they tied to a single reference mark which had evidently been disturbed; the original station mark was lost.

Results—The results computed from the final values are shown graphically in Figure 4. In that figure the analytically determined values of the constant error have been applied. The final tabulated results of the adjustment are shown in Table 3. The mean proportional error of about 9 parts in \( 10^8 \) is clearly compatible with the figure of \( \pm 1.4 \text{ km} \cdot \text{sec} \) derived as the probable error of the velocity determination from the computation, when it is considered that 15 measurements were made.

Recent published determinations of \( c \)—Esseen [1951] tabulated the values shown in Table 4. The writer believes that the value of \( 299,794.2 \pm 1.4 \) as published in this paper is far better than the previous value obtained from the earlier shoran work and that it should also be included in
Table 3--K and F corrections and residual errors (v's) derived from the adjustment by least squares

<table>
<thead>
<tr>
<th>Line</th>
<th>Ground station numbers</th>
<th>Shoran distance (s)</th>
<th>Shoran minus geodetic distance</th>
<th>Correction for K</th>
<th>Correction for F (F-1)s</th>
<th>v's from adjustment</th>
<th>Proportional error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-6</td>
<td>2 and 3</td>
<td>40.6041 mi</td>
<td>-0.0090 mi</td>
<td>+0.0057 mi</td>
<td>+0.0025 mi</td>
<td>+0.0006 mi</td>
<td>19.7 parts in 10^6</td>
</tr>
<tr>
<td>3-4</td>
<td>1 and 3</td>
<td>96.7049 mi</td>
<td>-0.0122 mi</td>
<td>+0.0039 mi</td>
<td>+0.0059 mi</td>
<td>+0.0024 mi</td>
<td>24.8 parts in 10^6</td>
</tr>
<tr>
<td>5-6</td>
<td>1 and 2</td>
<td>100.2986 mi</td>
<td>-0.0110 mi</td>
<td>+0.0056 mi</td>
<td>+0.0061 mi</td>
<td>+0.0007 mi</td>
<td>7.0 parts in 10^6</td>
</tr>
<tr>
<td>5-1</td>
<td>1 and 3</td>
<td>118.9840 mi</td>
<td>-0.0113 mi</td>
<td>+0.0059 mi</td>
<td>+0.0072 mi</td>
<td>+0.0002 mi</td>
<td>1.7 parts in 10^6</td>
</tr>
<tr>
<td>1-6</td>
<td>1 and 2</td>
<td>132.9865 mi</td>
<td>-0.0128 mi</td>
<td>+0.0056 mi</td>
<td>+0.0081 mi</td>
<td>-0.0009 mi</td>
<td>6.8 parts in 10^6</td>
</tr>
<tr>
<td>2-3</td>
<td>2 and 3</td>
<td>134.9546 mi</td>
<td>-0.0152 mi</td>
<td>+0.0057 mi</td>
<td>+0.0082 mi</td>
<td>+0.0013 mi</td>
<td>9.8 parts in 10^6</td>
</tr>
<tr>
<td>5-2</td>
<td>1 and 3</td>
<td>139.1127 mi</td>
<td>-0.0098 mi</td>
<td>+0.0039 mi</td>
<td>+0.0084 mi</td>
<td>-0.0025 mi</td>
<td>16.0 parts in 10^6</td>
</tr>
<tr>
<td>3-6</td>
<td>1 and 2</td>
<td>145.8276 mi</td>
<td>-0.0151 mi</td>
<td>+0.0056 mi</td>
<td>+0.0088 mi</td>
<td>+0.0007 mi</td>
<td>4.8 parts in 10^6</td>
</tr>
<tr>
<td>1-2</td>
<td>3 and 2</td>
<td>145.8768 mi</td>
<td>-0.0118 mi</td>
<td>+0.0057 mi</td>
<td>+0.0088 mi</td>
<td>-0.0029 mi</td>
<td>19.2 parts in 10^6</td>
</tr>
<tr>
<td>4-6</td>
<td>1 and 2</td>
<td>190.4889 mi</td>
<td>-0.0158 mi</td>
<td>+0.0056 mi</td>
<td>+0.0115 mi</td>
<td>+0.0013 mi</td>
<td>6.8 parts in 10^6</td>
</tr>
<tr>
<td>2-4</td>
<td>1 and 2</td>
<td>199.1735 mi</td>
<td>-0.0179 mi</td>
<td>+0.0056 mi</td>
<td>+0.0121 mi</td>
<td>+0.0002 mi</td>
<td>1.0 parts in 10^6</td>
</tr>
<tr>
<td>5-3</td>
<td>2 and 3</td>
<td>226.9698 mi</td>
<td>-0.0205 mi</td>
<td>+0.0057 mi</td>
<td>+0.0138 mi</td>
<td>+0.0010 mi</td>
<td>4.4 parts in 10^6</td>
</tr>
<tr>
<td>5-4</td>
<td>1 and 2</td>
<td>235.5042 mi</td>
<td>-0.0222 mi</td>
<td>+0.0056 mi</td>
<td>+0.0143 mi</td>
<td>+0.0023 mi</td>
<td>9.8 parts in 10^6</td>
</tr>
<tr>
<td>1-3</td>
<td>2 and 3</td>
<td>277.0347 mi</td>
<td>-0.0222 mi</td>
<td>+0.0057 mi</td>
<td>+0.0168 mi</td>
<td>-0.0003 mi</td>
<td>1.1 parts in 10^6</td>
</tr>
<tr>
<td>1-4</td>
<td>1 and 2</td>
<td>320.1271 mi</td>
<td>-0.0248 mi</td>
<td>+0.0056 mi</td>
<td>+0.0194 mi</td>
<td>-0.0002 mi</td>
<td>0.6 parts in 10^6</td>
</tr>
</tbody>
</table>

Means 0.00118 9.1 parts in 10^6

Table 4--Velocity of waves in vacuo (c)

<table>
<thead>
<tr>
<th>Author</th>
<th>Method</th>
<th>Velocity in vacuo</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Birge</td>
<td>Optical</td>
<td>299,776 ± 4</td>
<td>1</td>
</tr>
<tr>
<td>Essen and</td>
<td>Gordon-Smith Cavity resonator</td>
<td>299,792 ± 9</td>
<td>1</td>
</tr>
<tr>
<td>Bergstrand</td>
<td>Optical</td>
<td>299,793.1 ± 0.25</td>
<td>2</td>
</tr>
<tr>
<td>Aslakson</td>
<td>Shoran</td>
<td>299,792 ± 2.4</td>
<td>2</td>
</tr>
<tr>
<td>Essen</td>
<td>Cavity resonator</td>
<td>299,792.5 ± 3</td>
<td>2</td>
</tr>
<tr>
<td>Bol</td>
<td>Cavity resonator</td>
<td>299,799.3 ± 0.4</td>
<td>2</td>
</tr>
<tr>
<td>Resultant mean value</td>
<td></td>
<td>299,790.2</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4--Adjusted shoran results assuming a separate K for each ground station
any proposed value for c. In fact, the evidence now is very strong that the Birge statistical value should be abandoned entirely. A detailed examination of all of the results of the individual observers included in the determination of that value reveals a lack of consistency far below that obtained by recent observers. An average of the latest observations results in a new value of 299,782.2 which, in view of the better consistency of recent work, seems to be a reasonable figure to adopt for the present.

It can now be stated that, in shoran work in progress, a value of 299,793 km sec is being used in the computations and to date excellent checks are being obtained in comparison with surveyed distances. During the present work at least 9 to 11 new distance comparisons will be made.

References


U. S. Coast and Geodetic Survey,
Washington 25, D. C.

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