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Principles

It is a well known principle of geodesy that geographic positions of comparatively nearby points may be in error by several seconds, relative to one another when the positions are independently determined by astronomic observations. The importance of this fact has been greatly emphasized in the course of military mapping operations of the past few years, and it has become desirable to investigate various means of increasing the maximum effective range of geodetic control operations beyond that available by the use of conventional methods.

Such a method has been developed during the past year by the 311th Reconnaissance Wing, United States Army Air Forces, through the adaptation of a precise air navigation system, shoran, to the problem of geodetic control. Shoran enables the direct measurement, by electronic means, of distances up to 500 or 600 mi with geodetic accuracy. The equipment was originally designed to establish the position of an airplane by distance measurement to two ground stations of known position, the airplane having a maximum altitude of 40,000 ft and the corresponding maximum measured distances being about 300 mi. In geodetic operations, a revised procedure is followed. Shoran readings are used to determine the minimum sum of the distances from the airplane to two ground stations, and a scheme of triangulation is built up from these sum distances in much the same manner that conventional schemes are formed from the measurement of horizontal angles. It is apparent that by this procedure the effective range of shoran is doubled, since each of the measured distances may approach the maximum range of the equipment.

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To attain the greatest possible accuracy several refinements have been made in the original shoran operating methods. These refinements are fully described in a series of reports issued by the Seventh Geodetic Control Squadron of the 311th Reconnaissance Wing and will be treated briefly in this paper. During the course of the investigations, cooperation has been effected with the Division of Geodesy, United States Coast and Geodetic Survey, in the solution of certain problems, the most important of these being the derivation of a velocity correction formula and assistance in the development of new methods for the computation and least squares adjustment of triangulation schemes to be observed by shoran.

Basically, the shoran equipment consists of three main units, the airborne transmitter-receiver-indicator and the two ground station transponders. In operation, the airborne station transmits extremely short pulses at a rate of about 930 per sec, alternately on two different frequencies, the frequency change-over being effected by a vacuum relay at a rate of approximately ten per sec. These pulse groups are alternately received at the two ground stations and retransmitted (on a third frequency common to both) back to the airborne station, where they appear on a cathode ray indicator provided with a precisely controlled circular sweep which serves as the time base. A marker pulse is fed into the indicator directly from the pulse generator to serve as a reference for the transit-time measurement. Generation of the transmitted pulses is advanced by a series of goniometers or phase shifters until the returned pulses from both ground stations exactly coincide with the marker pulse on the screen. The amount of advance is then read from the goniometer dials in terms of distance to each of the ground stations. Lags in the various circuits and connecting cables are predetermined and inserted as corrections to the zero settings of the goniometer dials. The goniometers are calibrated in miles and read to a maximum accuracy of 1/1000 mi.

It is apparent that the accuracy of a geodetic distance as measured by shoran is dependent on three basic factors: (a) Accuracy of the measured time interval; (b) accurate knowledge of propagation velocity along the path; and (c) relationship between length of the propagation path and geodetic distance between the ends of the path. These factors will be considered in sequence.

Accuracy of the measured time interval--There are three basic elements in the accuracy of transit-time determination: (1) Time base frequency; (2) accuracy of the goniometers or phase shifters; and (3) inherent lags in the circuits and connecting cables. Errors in the time base frequency are negligible, the frequency being governed by a high-precision temperature-controlled oscillator. The goniometer errors have been found to be more serious in their effect on the final result. Theoretically, the phase advance produced by a goniometer is linear with respect to the angular rotation of the goniometer dial, but manufacturing tolerances are such that a deviation from linearity of several degrees may occur. To illustrate the effect of this error, it is pointed out that a deviation of 4° produces an error of about 50 ft in the dial reading. This error may be reduced by making the observations in such a manner that different goniometer settings are used in successive distance measurements, a procedure which may be compared to the established practice of using different circle settings in the measurement of horizontal angles with a theodolite. Errors introduced by uncertainties in lag of the various circuits and cables tend to be systematic in a given installation, and careful adjustment is required to minimize this effect.

Accuracy of propagation velocity--The shoran dials are calibrated to read correct path distance assuming the propagation velocity to remain constant at its sea-level value along the entire path. Since the path may extend to altitudes of 30,000 ft or more, it is evident that the decrease of air density and moisture content with altitude produces a significant effect on dielectric constant and consequently on propagation velocity. To illustrate, this change in dielectric constant results in a velocity correction of about 50 ft in the case of a plane altitude of 25,000 ft and a shoran distance of 200 mi. A formula has been developed for determination of the velocity correction as a function of plane altitude, ground station altitude, and shoran distance. The constants of this formula are dependent on the dielectric constant-altitude relation existing in the NACA Standard Atmosphere, modified to include a mean moisture lapse rate. Actual soundings taken in various regions of the United States indicate that this standard atmosphere may generally be used, although in certain cases the accuracy of the formula is increased by computing constants for several type atmospheres and using that type which most nearly approximates the soundings available at the time and place of observation.

The accuracy of propagation velocity along the path is, of course, directly related to the accuracy of the sea-level velocity used in calibrating the shoran timing unit. The basic physical value now used is the velocity of light in a vacuum, which after reduction to standard sea-level dry air conditions gives a calibration velocity of 186,218 mi/sec. The precision of this value is uncertain, but is probably about one part in 50,000. It is anticipated that recent developments in micro-wave technique will make possible the accurate determination of radio propagation velocity directly by phase comparison over a precisely measured base, and thus decrease uncertainty in the velocity constant.

Relation between path length and geodetic distance--The shoran path is assumed to be a circular curve extending from the ground station having an altitude K to the airborne station at altitude H, the path curvature being a result of the decrease of refractive index with respect to altitude. If the refractive index gradient were constant, the shoran ray path would be a circular curve with radius depending upon the amount of gradient. Actually the gradient follows a generally decreasing trend as the altitude is increased, resulting in an increase in path radius. Experience has shown that a mean path radius may be adopted for the geometrical correction formula. Having determined this radius value, the reduction to sea-level geodetic distance M is accomplished by a series of four corrective terms as shown below.

$$\begin{aligned} M = S &- (2.3920 \times 10^{-8})[S(H + K)] - (1.7935 \times 10^{-8})[(H - K)^2/S] \\ &+ (0.24848 \times 10^{-8})S^3 - (1.6083 \times 10^{-16})[(H - K)^4/S^3] \quad \ldots\ldots\ldots\ldots (1) \end{aligned}$$

where H and K are in ft and M and S are in mi. It is interesting to note that the terms of formula (1) bear a significant resemblance to certain of the corrections commonly used in geodetic surveying. The first term suggests the correction used in reducing a measured base to its sea-level value; the second and fourth are analogous to those used in correcting a catenary for slope; and the third term provides the rectification from path curvature to earth curvature. The coefficient of the third term contains the earth radius and path radius, both assumed to be constant. The fourth term is rarely required, being negligible unless the slope of the path exceeds one part in ten.

In this derivation of the geometrical formula, certain assumptions have been made for the purpose of simplification. Errors resulting from these assumptions have been evaluated, and tables and curves prepared to facilitate modification of the formula when required. Considering the observational accuracy available in present shoran equipment, formula (1) is used as given, but it was decided to make the refinements available for future use when warranted by improvements in accuracy.

The geometrical reduction requires accurate knowledge of the plane altitude H. Investigation has shown the effect of altitude errors to be most serious when the path length is less than 100 mi; at greater distances the length error remains fairly constant for any given altitude error and has a magnitude of about ten ft for each 200 ft of error in altitude. Use of a pressure altimeter alone may result in an altitude error of several hundred ft in certain cases where meteorological variations are large, although the error may be minimized by careful analysis of available radiosonde data. Altitude accuracy is increased by use of the radio altimeter as a calibration control for the pressure instrument; during operations over water surfaces the radio instrument may be used exclusively. It is believed that by following these procedures, an altitude tolerance of 50 ft or less can be maintained during shoran operational flights.

Method of observation

To measure distance between the two ground stations, A and B, the airborne station is flown at constant ground speed and altitude on a projected course approximately normal to the line AB at a point about midway on the line. The shoran operator maintains the two ground station pulses in constant alignment with the marker pulse on the indicator screen by rotating the goniometer dials. The dial readings are continuously photographed by a motion picture camera at a rate of one frame every three seconds, thus providing a permanent recording of the observations. Each frame records two shoran distances, one to each ground station, the frame number, k, being linearly related to distance along the flight path. The relation between the sum distance, S, and the frame number, k, is a hyperbola with constants depending on the plane altitude and the approximate distance between the ground stations. In practice it has been found desirable to substitute a parabola for the hyperbola in order to simplify the computation; this may be done without significant error since the minimum values of the two curves are almost identical. The general form of the parabolic expression is

$$S - ak^2 + bk - c = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where S = sum of two shoran distances on any frame, k = frame number, and a, b, and c are constants to be determined.

Since the S values contain observational errors, the most probable values for constants a, b, and c are determined by least squares using an observation equation for each frame. Having determined these constants for any single line crossing, the expression is differentiated and equated to zero.

$$dS/dk = 2ak - b = 0$$

whence

$$k = b/2a \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

The value for k given by (3) is substituted in expression (2) and the corresponding minimum value for S determined. It is pointed out that k is ordinarily a fractional quantity in this operation. To find the geodetic distance between the two ground stations, the integral value of k closest to that given by (3) is used and the shoran distance S_1 to one of the ground stations is read from this frame. The shoran distance S_2 to the second station is then obtained from $S_2 = S_{min} - S_1$. The values S_1 and S_2 are used, together with the ground station elevations and the airplane altitude, to obtain the geodetic distances from the plane to each ground station by the geometrical formula previously described. The sum of these reduced distances is then the observed geodetic distance between the ground stations.

In making the observations, the use of about 15 frames on either side of the minimum point, or 31 in all, has been found satisfactory. By standardizing the number of frames used, certain elements in the least squares solution may be pre-computed and used for all line crossings with a considerable saving in computing time. After the mathematical solution has been made, it is general practice to plot the S value recorded on each frame against the parabola obtained by the least squares solution. This provides a visual representation of the residual obtained for each frame and permits more accurate decisions in cases of possible rejections. A typical minimum-distance curve is illustrated in Figure 1. In this example, the ordinate of the graph represents increment of the sum distances above a reference value of 227.540 mi. The reference value selected is always slightly less than the minimum recorded distance, for convenience in computation and plotting. Several crossings of an individual line are made, and the mean value used as the geodetic distance between ground stations.

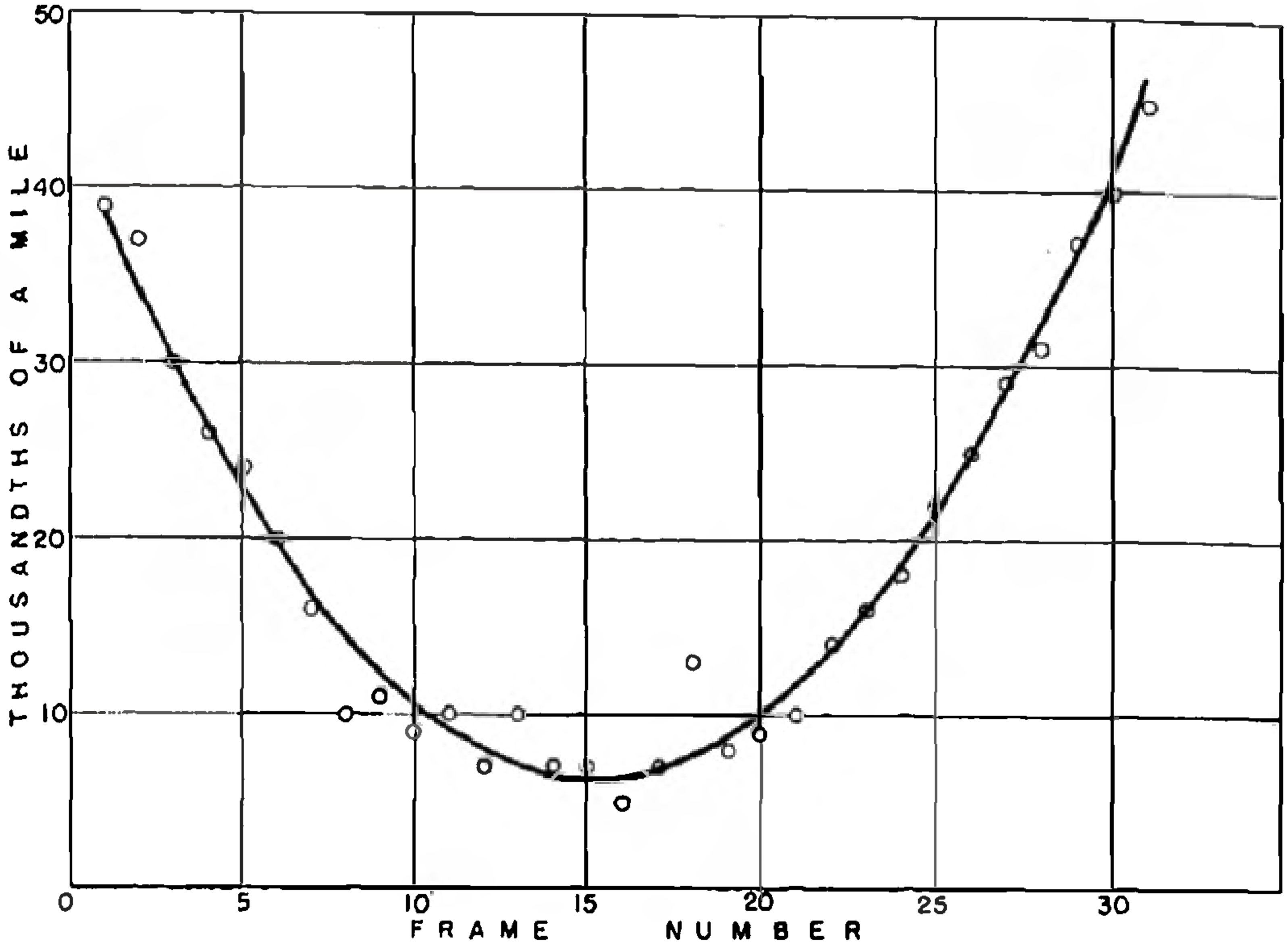

Fig. 1--Shoran line crossing minimum distance curve, Cheyenne-La Junta, May 1, 1946

Computation and adjustment

Revised procedures are required in the computation and adjustment of shoran triangulation. The observed lines may approach 500 or 600 mi in length and many of the assumptions and approximations used in conventional triangulation introduce errors of considerable magnitude in the final results. A new method of least squares adjustment is required in view of the fact that the basic observational data are geodetic distances rather than horizontal angles.

The basic figure used is a triangle whose measured sides are geodetic distances from which the corresponding spheroidal angles may be determined by application of LEGENDRE'S Theorem, provided certain refinements are used in the computation. The first step is the determination of the angles of a plane triangle having sides equal to the geodetic values; any of the standard formulas of plane trigonometry may be used for this operation. Spherical excess is next computed by the usual formula, with the addition of one corrective term, and distributed unevenly among the three angles in accordance with the formula by CLARKE [1880].

$$A_p - A_s = -\epsilon''/3 - [\epsilon''(-2a^2 + b^2 + c^2)]/180R_m^2 \quad \ldots\ldots\ldots\ldots\ldots (4)$$

where A_p = plane angle, A_s = spherical angle, ϵ = spherical excess of triangle, R_m = mean radius, and a, b, and c are the triangle sides.

Having determined the spherical angles, a correction may be applied to each angle to reduce to its spheroidal or geodesic value. This correction is given by TOBEY [1928].

$$\tau_A'' = -[\sin(\phi_m + \phi_A)\sin(\phi_m - \phi_A)\epsilon'' e^2]/6 \quad \ldots\ldots\ldots\ldots\ldots (5)$$

where $\tau = A_s - A$, ϕ_m = mean latitude of triangle, and e^2 is the minor eccentricity.

The above refinements provide a determination of the geodesic angles within a tolerance of 0.01 sec in a triangle having maximum length of side of 600 mi. In the case of shorter sides and/or greater tolerance in computational error, the corrections given by (4) and (5) may often be neglected.

The precise determination of geographic positions over distances of 500 or 600 mi presents great difficulty from the viewpoint of rapid and accurate computation. The method given by HELMERT, while providing accurate results, is rather forbidding in its complexity, and investigation is under way as to alternate methods with emphasis on applicability to routine computing.

Three methods of accomplishing the least squares adjustment of a shoran scheme have been proposed. These methods are quite different in approach and each has proved satisfactory in computational tests. No decision has been made as to the final method or methods to be adopted, pending further study of practical considerations involved. When a quadrilateral is determined by direct measurement of its sides only, one condition equation results as contrasted with four in the case of horizontal angle measurement. Considering one side of the quadrilateral to be fixed in position, azimuth, and length, the ratio of conditions per new station determined is thus seen to be one to two when the sides alone are measured. This ratio is improved by the substitution of a five-sided figure with all diagonals observed; the condition-station ratio then becomes one to one, making the figure comparable in strength to the conventional single triangle with all angles observed. Accordingly, the five-sided figure has been adopted as the basic unit in the layout of proposed triangulation.

Results of accuracy tests

Table 1 gives results obtained during recent field tests conducted by the 311th Reconnaissance Wing in Colorado, Kansas, Nebraska, and Wyoming. Ground stations were established at United States Coast and Geodetic Survey first-order triangulation points, and geodetic distances obtained by inverse computation were used to evaluate accuracy of the shoran measurements.

Table 1--Results of shoran accuracy tests in Colorado, Kansas, Nebraska, and Wyoming, 1946

Geodetic line	Number of crossings	Observed distance	True distance	Observed minus true distance	Proportional error
		mi	mi	mi	
La Junta-Garden City	18	148.5322	148.5395	- 0.0073	1/ 20,300
Cheyenne-Imperial	17	173.7443	173.7471	- 0.0028	1/ 62,100
Imperial-Garden City	17	181.3687	181.3694	- 0.0007	1/ 259,100
La Junta-Imperial	44	198.7188	198.7099	+ 0.0089	1/ 22,300
Cheyenne-La Junta	47	227.2917	227.2868	+ 0.0049	1/ 46,400
Cheyenne-Garden City	19	308.5250	308.5252	- 0.0002	1/1,542,600

It is difficult at this stage to estimate the ultimate accuracy to be obtained by the shoran method. The equipment was originally designed to produce a degree of precision somewhat lower than that required in geodetic operations, and although many refinements have been made to date, further progress is anticipated. As the ultimate in instrumental accuracy is approached, propagation uncertainties will become better defined in magnitude and may well prove the limiting factor in distance determination by shoran.

References

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