Erik BERGSTRAND

MEASUREMENT OF DISTANCES BY HIGH FREQUENCY LIGHT SIGNALLING

Summary

A source of light emits light, the intensity of which varies with a frequency which is kept constant by means of an oscillating quartz crystal. A plane mirror is placed at a distance of 10-30 km from the source of light. The mirror reflects the light back to a phototube placed in the neighbourhood of the source of light. The above-mentioned crystal controls the sensitivity of the tube, which thus varies with the same frequency as the source of light. Thereby, the electrical currents from the tube will vary with the distance, depending on whether the incoming light impulses are more or less timed to the cycles of sensitivity of the phototube. Thus the variation of the current is periodic with the distance. With the actual rate of crystal frequency, the same strength of current is repeated every 18th meter that the mirror is moved. When measuring distances the periodical change with the distance is employed as a scale. The size of the scale is once and for all determined from a known distance. A special arrangement causes the currents to reverse every 9th meter of change in distance. Thus the currents pass through zero and have their greatest rate of change in relation to a change in distance. The 0-points represent the division-lines of the scale of distance used. The constancy of the distance between the division-lines is directly dependent on the constancy of the frequency. The determination of a distance is accurate to about one in one million.

In triangulations of the first order we now and then have to determine the length of the side of a triangle. Usually the length of the side is 30 km and nowadays the determination of the length is almost exclusively effected by measuring a base by means of invar tapes and the enlargement of the base to the required side. The accuracy of the determination is about 1 : 500 000.

A method of directly measuring a side of 30 km is to make use of the circumstance that the velocity of light has a finite and, as it seems, invariable value. The first principle that suggests itself is the one that has been employed ever since the attempts by Fizeau in 1849 to make terrestrial determinations of the velocity of light. Adapted to the measurement of lengths the principle is expressed as follows.

A source of light emits light, the intensity of which varies with a given frequency. The light is reflected back from a distant mirror and observed by an indicator, the sensitivity of which varies with the same frequency as that of the emitted light. The frequency being constant, the reaction of the indicator varies with the distance to the distant mirror. This cyclic variation with the distance provides a scale which can be employed for measuring distances, when once the size of the scale has been determined.

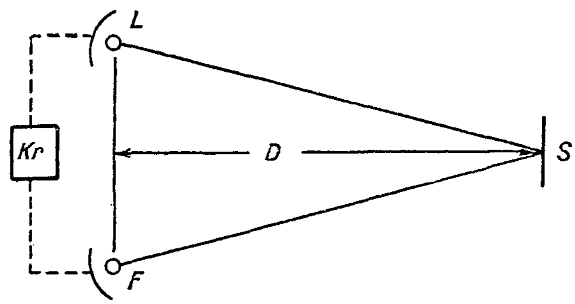

Fig. 1

In Fig. 1 we see an arrangement of the apparatus. L is the source of light having a spherical mirror to collect the light in a beam. The source is influenced by high frequency tension from the crystal-controlled oscillator Kr and thus the intensity of the emitted light varies with the frequency of the oscillator. S is a plane mirror which reflects back the light to the phototube F. The phototube gets its running tension from Kr. Accordingly the sensitivity of the tube varies with the same frequency as the emitted light. As the rapid blinks of light spend a certain time to cover the way to S and back again, the moments of high sensitivity of the tube will be more or less timed to the incoming blinks, depending on the distance D. Thereby the currents taken from the tube will vary with the distance D. Maximum of current will occur at certain definite distances $D = N \cdot D_n$, where $N =$ a whole number and D_n depends on the frequency n. The maximum points could represent the division-lines of a scale, intended for measuring distances. However, the maximum points are not sharply defined. Then suppose we had two identical apparatus similar to that in Fig. 1, the one giving maximum of current at the same time as the other giving minimum of current at $D = N \cdot D_n$. Having an instrument measuring the difference of the currents we then get 0-current half way between a maximum and a minimum. The variation of the current here being large, the 0-points would be sharply marked and thus more adapted to constitute the scale. Instead of two apparatus we could have a single one, being influenced by a low frequent tension in such a manner that it would give maximum of current during the positive half-cycles and minimum of current during the negative half-cycles (at $D \doteq N \cdot D_n$). Thus in the former case the two apparatus are separated with regard to space, in the latter case with regard to time.

At Lovö, the geodetical station of Rikets Allmänna Kartverk, some practical trials have been made with an apparatus according to the last-mentioned principle. In Fig. 2 we see a summary layout. L is the source of light, a 30-watt incandescent lamp, S_1 and S_2 are 45-cm spherical mirrors, S_3 a plane mirror, 1, 2, 3, 4, 5, LS_1 and LS_2 are lenses and mirrors, F is a vacuum phototube and I an instrument

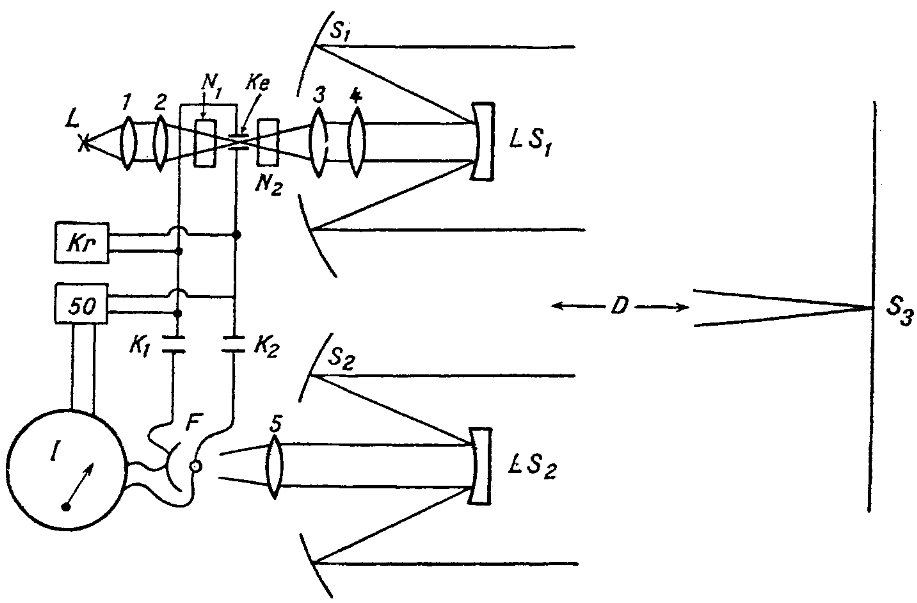

Fig. 2

on which the strength of the current from F is read off. Ke is a Kerr cell. It consists of a pair of condensator-plates immersed into nitrobenzene. If the plates have different electrical potentials, the liquid in the field between the plates get double-refracting properties, the planes of polarisation being parallel and perpendicular to the direction of the electrical field. The light coming into the space between the plates is polarized by the Nicol prism N_1. The plane of polarisation is inclined 45° to the direction of the electrical field. Between the plates, the light oscillation in the N_1-plane is divided into two equally sized parts. The one part includes oscillations parallel to the field, the other those perpendicular to the field. The propagation velocities of the two classes of oscillations differ from each other. Thus departing the space between the plates the one class is a part of a wavelength of light in front of the other. The magnitude of this part is proportional to the square number of the difference of tension (of course, the tension being sufficiently high, the parts will turn to whole wave-lengths). Because of the arising difference of phase between the two classes of oscillations the light now is elliptically polarized, the excentricity varying between 0 and 1, depending on the tension. The light is analyzed by the prism N_2, the plane of polarisation of which is perpendicular to that of N_1. Outside the prism N_2 the intensity of the light depends on the tension in accordance with the equation :

$$J = J_0 \cdot \sin^2 (k \cdot V^2) \qquad (1)$$

where J = intensity, J_0, k = constants. V = difference of potential between the plates.

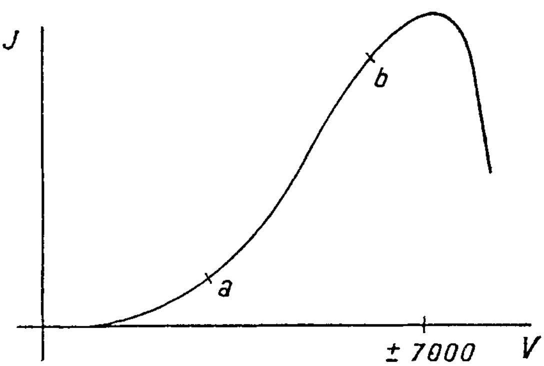

Fig. 3

The intensity of the light as a function of the tension is shown by Fig. 3. Red light and the distance between the plates = 2 mm. Between a and b the curve is fairly straight. If the cell has a tension of rest between these points and an alternating tension of limited amplitude superimposed, the variations of the intensity of light assume the same shapes as the alternating tension + a quantity of constant light, the latter depending on the tension of rest. The plates in the Kerr cell are fed by the crystal oscillator *Kr* with a high frequency tension (8.33 · 10^6 cycles/second) and the amplitude 1 200 volts and also from the net transformer *50* with 50-cycle tension having the amplitude 6 200 volts.

The 50-cycle oscillation being square in shape (⊓⊔⊓⊔), we attain the most favourable condition. The result would be a completely constant tension of rest, changing sign 100 times a second. Besides, such a shape would more strictly suit the following analysis. A divergence from the square shape however does not change anything in the point of principle in the analysis. Later on, probably, a square formed tension will be used, the advantages being considerable.

The high frequency has purely sine form. Arrived at the plates of the Kerr cell this tension is assumed to have the form :

$$V = a \cdot \sin \omega t \qquad (2)$$

a = amplitude, $\omega = 2\pi n$, where n = frequency, t = time.

It takes a short time t_1 to transform this tension into a corresponding light variation immediate outside the prism N_2. The light variations will there have the form :

$$J_{50+} = C_1 + C_2 \cdot \sin \omega \, (t - t_1). \tag{3 a}$$

C_1 is the (relatively) constant intensity and C_2 the amplitude of the variations. 50^+ indicates that the flow of light is emitted during the positive half-cycles of the 50-cycle alternation. Thus we have supposed $t = 0$ to occur during a positive half-cycle. Then during the negative half-cycles the intensity will have the form :

$$J_{50-} = C_1 + C_2 \cdot \sin \left[\omega \left(t - t_1 \pm K \cdot \frac{1}{n}\right) + \pi\right] \tag{3b^1}$$

K is so large a whole number, that $K \cdot 1/n$ will reach within a negative half-cycle from $t = 0$ ($\cong 0.01$ sec.). As $1/n = 2\pi/\omega$ we get an angle of phase of $K \cdot 2\pi$ and J_{50-} may be written :

$$J_{50-} = C_1 + C_2 \cdot \sin \left[\omega \, (t - t_1) + \pi\right] \tag{3 b}$$

The angle π appears in one of the equations (3 a) and (3 b) because of the high frequent oscillations being displaced 180° every time the 50-cycle tension changes its sign, the electrodes of the cell then changing polarity.

Having passed the distance D to the plane mirror S_3 and back again, the light (or a part of it) is absorbed by the phototube F. This is a multiplicating tube of the shape shown by Fig. 4. K is the cathode against which the light is cast. A is the anode. 1—9 are called binodes. The binode 9 is earthed and its tension is zero. Then the tension

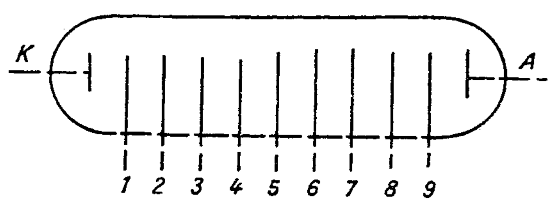

Fig. 4

increases by —90 volts for every further electrode on to the cathode. On the anode is put a purely high frequently alternating tension, taken from the plates of the Kerr cell. The 50-cycle tension is stopped by the condensators K_1K_2. When the cathode is illuminated, a number of electrons are freed and rush towards the binode 1. At their impact each of them frees three or four fresh electrons, which rush towards binode 2 and so on up to 9, where a large number, proportionally to the illumination, are free to be absorbed by the anode at the moments suffered a loss of time t_2 compared with the illumination of the when this electrod has positive tension. The intensity of the light thrown upon the cathode, varies according to :

$$J_{50+} = C_3 + C_4 \cdot \sin \omega \left(t - t_1 - \frac{2D}{c} \right) \tag{4 a}$$

$$J_{50-} = C_3 + C_4 \cdot \sin \left[\omega \left(t - t_1 - \frac{2D}{c} \right) + \pi \right] \tag{4 b}$$

In the time quantity $2\ D/c$, where $c =$ the velocity of light, $2\ D$ is the whole distance from the prism N_2, or, more exactly, from the point where t_1 in (3) was reckoned, and to the mirror S_3 and exactly the equal distance back again. The cathode is imagined as being placed here. If it was not, it would be necessary to add a further time quantity $t_k = d/c$, where d is the distance of the cathode from the end of the length D from the mirror S_3.

The magnitude of the quantities of electricity in form of space charges, which the illumination makes available at binode 9, has suffered a loss of time t_2 compared with the illumination of the cathode, because of the running time between the different electrodes. Thus momentarily the magnitude of the charges has the form :

$$Q_{50+} = A + B \cdot \sin \omega \left(t - t_1 - t_2 - \frac{2D}{c} \right) \tag{5 a}$$

$$Q_{50-} = A + B \cdot \sin \left[\omega \left(t - t_1 - t_2 - \frac{2D}{c} \right) + \pi \right] \tag{5 b}$$

Q_{50+} and Q_{50-} appear as currents from the anode during the positive high-frequent half-cycles of this electrode. A time t_4 arises in the connection between the plates of the Kerr cell and the anode, where the tension has the form of eq. (2), and a positive half-cycle will begin at the time t_4 and end at $t_4 + {}^1/_2\ n$, where $n =$ the frequency. If we sum up Q_{50+} and Q_{50-} during this time, we get the current formed by the respective series of 50-cyclic half-cycles. Besides, the current being the quantity of electricity per second, we have to multiply by $n/2$. With $n = \omega/2\pi$ this will give :

$$i_{50+} = \frac{1}{2} \cdot \frac{\omega}{2\pi} \int_{t_4}^{t_4 + \pi/\omega} \left[A + B \cdot \sin \omega \left(t - t_1 - t_2 - t_3 - \frac{2D}{c} \right) \right] \cdot dt \tag{6 a}$$

$$i_{50-} = \frac{1}{2} \cdot \frac{\omega}{2\pi} \int_{t_4}^{t_4 + \pi/\omega} \left[A + B \cdot \sin \omega \left(t - t_1 - t_2 - t_3 - \frac{2\,D}{c} \right) + \pi \right] \cdot dt, \tag{6 b}$$

where t_3 denotes the running time of the electrons from binode 9 to the anode. Eq. (6 a, b) give :

$$i_{50+} = \frac{A}{2} + \frac{B}{2\pi} \cdot \cos \omega \left(t_4 - t_1 - t_2 - t_3 - \frac{2D}{c} \right) \qquad (7\ a)$$

$$i_{50-} = \frac{A}{2} - \frac{B}{2\pi} \cdot \cos \omega \left(t_4 - t_1 - t_2 - t_3 - \frac{2D}{c} \right) \qquad (7\ b)$$

In Fig. 2 it is seen that the instrument *I* is influenced by the source of 50-cycle alternating tension (*50*). This is effected by the coupling shown schematically in Fig. 5. The grids *a* and *b* of the valves 1 and 2 are fed by the 50-cycle tension in such a way that *a* has a maximum of positive tension at the same time as *b* has a maximum of negative tension. During the positive half-cycles, previously denoted by 50^+, the grid *a* has positive tension and valve 1, working normally, supplies current to I proportionally to i_{50+} of eq. (7 a). During this half-cycle valve 2 is totally blocked by the high negative tension on the grid *b*, the current through this valve being

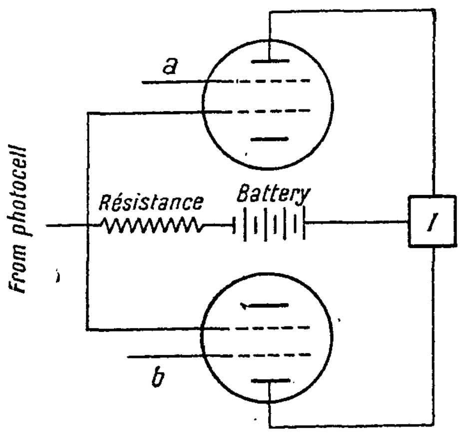

Fig. 5

$= 0$. During the half-cycles denoted by 50^- the part of the valves 1 and 2 are interchanged and the current is reversed through *I*. This is a direct current instrument reacting slowly and thus showing the difference between i_{50+} and i_{50-}. This gives :

$$i = i_{50+} - i_{50-} = \frac{B'}{\pi} \cdot \cos \omega \left(t_4 - t_1 - t_2 - t_3 - \frac{2D}{c} \right) \qquad (8)$$

where B' may have a value diverging from B, depending on intervening amplification. The current will be $i = 0$ to values on D which fulfill the condition .

$$\omega \cdot \left(t_4 - t_1 - t_2 - t_3 - \frac{2D}{c} \right) = \frac{\pi}{2} - N \cdot \pi, \qquad (9)$$

where N denotes a whole number with an arbitrary sign. With $\omega = 2\pi n$ and $c = \lambda \cdot n$, where $\lambda =$ the wave length, we get :

$$D = \frac{n}{2}\,(t_4 - t_1 - t_2 - t_3) \cdot \lambda + \frac{2N-1}{8} \cdot \lambda, \tag{10}$$

where $n = 8.33 \cdot 10^6$ and $\lambda = 36.0$ m.

Eq. (10) may be written :

$$D = K + \frac{2N-1}{8} \cdot \lambda, \tag{11}$$

where

$$K = \frac{n}{2}\,(t_4 - t_1 - t_2 - t_3) \cdot \lambda. \tag{12}$$

In eq. (11) there is no amplitude. This means that when the instrument shows zero D is independent of the intensity of the light. Flickering of the light due to optically turbulent air will not disturb the measurement. For the rest, a zero indication is defined by the instrument showing unchanged deviation whether the phototube is illuminated or not.

If we know the respective t-values we can compute the magnitude of K in eq. (11). t_1 is the reaction time of the Kerr cell along to the front of the prism N_2. t_1 can be estimated at :

$$t_1 = 0.05 \cdot 10^{-8} \text{ sec.} \tag{13}$$

The magnitude of $t_2 + t_3$ can be calculated from the dimensions of the phototube. The electrons there have to run through 10 successive lenghts of 0.7 cm and a difference of tension of —90 volts, the velocity of the electrons being $= 0$ at the beginning of each length. Putting

the charge of the electron	$e = 4.8 \,.\, 10^{-10}$		e. s. u.
» mass » » »	$m = 9 \,.\, 10^{-28}$		gram
tension run through	$V = 90/300$		e. s. volts
distance » »	$S = 0.7$		cm
field strength	$F = \dfrac{V}{S}$		

we get the force acting on the electron :

$$p = \frac{d^2 s}{dt^2} \,.\, m = F \,.\, e \tag{15}$$

By integrating (15) we get :

$$= \sqrt{\frac{2 \,.\, m \,.\, s}{F \,.\, e}} = 2.4 \,.\, 10^{-9} \text{ sec.} \tag{16}$$

As the number of lengths was 10, we get

$$t_2 + t_3 = 2.4 \,.\, 10^{-8} \text{ sec.} \tag{17}$$

t_4 depends on the condensators $K_1 K_2$ (fig. 2). Their value is 3.0 $\mu\mu F$, and they have loads of 20.000 Ω and 30 $\mu\mu F$ in parallel. If $n = 8.3 \cdot 10^6$ this will give a phase angle of 1°45′ or

$$t_4 = -0.05 \cdot 10^{-8} \text{ sec.} \tag{18}$$

Thus eq. (12) :

$$K = \frac{1}{2} \cdot 8.3 \cdot 10^6 \cdot (-0.05 - 2.4 - 0.05) \cdot 10^{-8} \cdot 36.0, \tag{19}$$

$$K = 3.7 \text{ m.}$$

With this value of K we get positive values of D by eq. (11) if $N = +1, 2, 3, 4, \ldots$

The first time the instrument shows $i = 0$ is, by (11), at the distance

$$D_1 = 0.8 \text{ m.} \tag{20}$$

0.8 m is mere approximate. By direct measurement we get

$$D_1 = 0.9 \text{ m.}$$

After that we get $i = 0$ every 9th meter that D increases. At alternate points the current according to (8) passes through zero from plus, at alternating points through zero from minus values. Every 18th meter the same zero point recurs.

The zero points marking the successive distances of the mirror S_3 from the Kerr cell and the photocathode, constitute the scale fixing the required distances. The starting-point of the scale is the first zero point at the distance $D_1 = 0.9$ m. By means of a variable loop of light D_1 can be determined or controlled. The variation is inconsiderable and generally negligible. For the rest the scale is constant to the same degree as $\lambda = c/n$ is constant. c is the velocity of light. This quantity varies somewhat, depending on atmospheric conditions. The variation is $0.9 \cdot 10^{-6}$ pr degree C and $0.4 \cdot 10^{-6}$ pr mm Hg of air pressure. n is the frequency of the crystal, which is practically only dependent on the temperature. The variation is maximum 15 cycles/degree. A thermostat keeps the temperature constant within ± 0°.05 C.

In the case of actual measurements it is impracticable to move the mirror backwards and forwards until the instrument indicates 0. Instead of that the frequency n is changed by a small known amount. The coupling scheme of fig. 6 shows how this can be done.

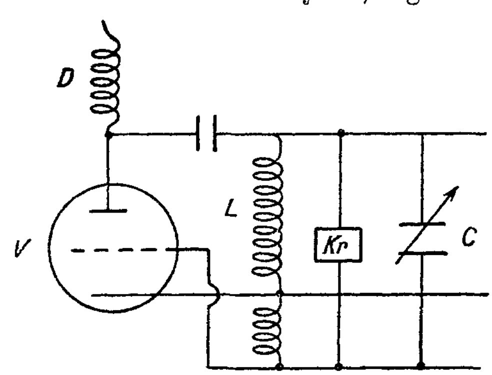

Fig. 6

Kr = the crystal, C = variable capacity, V = valve, L = throttle.

The frequency of the circuit varies with C according to the curve shown in fig. 7. The dotted line shows the relation between C and n if the crystal were not there. When the crystal is there, the curve bends asymptotically along the crystal frequency as the C decreases. This continues until the point a is reached, when the frequency suddenly jumps into the upper branch of the curve. If C is now increased, the curve bends in a similar manner towards the point b, where the frequency falls to the lower branch. Close to the points a and b, the frequency is only very slightly influenced by variations in capacity, self induction or working tensions. Point a is the working point. Here the variable capacity is completely turned out with the scale of the

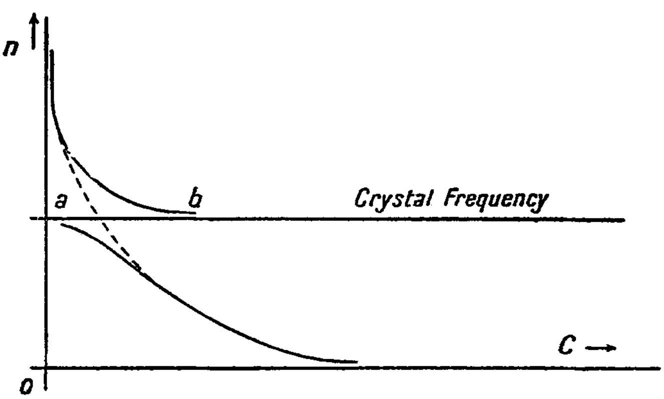

Fig. 7

wheel at zero. This scale is evaluated according to the change of frequency which occurs as C increases. To begin with the frequency decreases by 15 cycles for each point on the C-scale. The reading off can be estimated to within 2 cycles. At $D = 18.000$ m, including 500 λ, a change of 2 cycles of n means a displacement of 4 mm of the extreme 0-point.

Testing with $D = 10$ km.

Distance: Lovö-Varby = 7.734 m.

Reading off of the crystal scale :

33.2	Atm. pr.: 773 mm Hg
32.8	
32.9	Temp.: ± 0° C
33.0	

Mean: 33.0 ± 0.09 = 510 cycles/sec.

Thus 510 indicates the necessary change of frequency from the normal value (with crystal scale at 0) to bring the extreme 0-point to coincide with the mirror S_3. With $D = 7.734$ this means a displacement of the 0-point of

$$d = 0.48 \pm 0.0014 \text{ m at } P = 773 \text{ mm}, T = 0^\circ \text{ C}.$$

Visibility good. The adjustment of the mirror was constant.

Corresponding tests on the distance Lovö-Masmo = 11.025 m resulted in the following series of readings :

33	38	34.5	35.5	$P = 775$ mm
33.5	37.5	41	40.5	
37	34	38	36.5	$T = +3^\circ$ C

$$d = 0.86 \pm 0.03 \text{ m at } P = 773 \text{ mm and } T = \pm 0^\circ \text{ C}.$$

Visibility not really good. The plane mirror had to be continually adjusted.

The next night, with the miror in the same position, the readings were :

33	$P = 772$ mm
35	
36	$T = +4^\circ$ C
41	

$$d = 0.88 \pm 0.06 \text{ m at } P = 773 \text{ mm and } T = \pm 0^\circ \text{ C}.$$

Visibility poor.

The coordinates indicate the Masmo distance to be 11.024,77 m at « normal » frequency (from first to last 0-point).

Knowing the approximate values for frequency and the velocity of logt, we get 613 18-meter-lengths. Thus the unit length is 17.98494 m. Using this value we get the Varby distance as 7.733,52 m. The same distance from the coordinates is 7733,73 m. This in within the limits of error for the triangle points employed. More exact agreement is possible only with base lines.

Mr G. Bilius gave invaluable help during the tests by adjusting the distant plane mirror.

Results of the Lovö tests.

The accuracy of the adjustment is better than 1 : 1.000.000 at a distance of 10 km.

Good visibility is necessary.

At least twilight is essential for the determinations.

With good visibility a measurement can be completed within two hours, including the adjustment of the instruments.

Discussion of the tests.

The apparatus was constructed for an infra-red sensitive photocube. This tube, however, was destroyed and a blue sensitive tube was hastily inserted without any adaption. The quality of the tube was moderate (R. C. A. 1 P 22). The effect of the light source was only used to 25 % (no overloading). Four successive mirrors were gold plated. Theoretically, a high-class tube (1 P 21) and an adapted apparatus would yield an effect about 100 times greater. Thus it may unhesitatingly be claimed that the same results could be attained with $D = 30$ km.